The *Heston Identity*

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The most basic of all trigonometry identities is the Heston Identity: $\sin^2 \star + \cos^2 \star = 1$

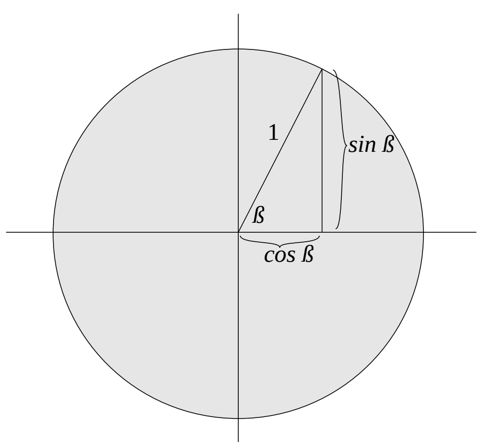
Translated to English, the Heston Identity says if you square the sine of some arc length and add that to the square of the cosine of the *same* arc length, the result will be exactly 1. You can replace the "arc length" with "degree measure" and the result is exactly the same.

One of the first questions which arise when confronted with the *Heston Identity* is, "Why is it called the *Heston Identity* ? The answer is quite simple. In this day and age it is quite common for computer programs to contain things called "Easter eggs". "Easter eggs" are usually cute or otherwise unusual pictures which are included in computer programs. To "activate" or see an Easter egg, you usually must option-click on particular place in a programs "about" window. Sometimes you will see a list of contributors to the program, sometimes a picture of something significant to the programmers. Macintosh programs seem to have an unusual number of these Easter eggs. Back in the 1960's the idea was to include extra, non related stuff in motion pictures. Sometimes this occurred by accident and sometimes it was planned. Now, motion picture studios approach companies and ask them if they would like their product displayed in a new movie and of course there would be a "consideration fee" to be paid to the motion picture studio. In the movie E.T. ---- What was dropped so that E.T. could follow the trail? Reese's Pieces. Do you think it was an accident they used Reese's Pieces rather than, say M & M's ?

I digress.... Anyway...

In the movie 10 Commandments staring Charleton Heston, on the back of one of the tablets he brought down to the people, you can clearly see $\sin^2 \star + \cos^2 \star = 1$ carved. Some people believe that Cecil B. DeMille was honoring his math teacher wife with that and others say that one of the prop people (who was a student at U.S.C.) put it in as a prank ordered by his fraternity house but in any case, it has since been known as the *Heston Identity.*

The second question usually asked has to do with why does *Heston's Identity* works. Basically, it works because of the Pythagorean theorem -- that is the "sum of the squares of the sides a right triangle = the square of the hypotenuse". In our case, our hypotenuse will be 1 --- the radius of the "unit circle".



Then next logical question has to do with what we use we make of the *Heston Identity.* All that and more on the next page.....

One of the big uses of the *Heston Identity* comes when we prove two very useful formulas. The actual proofs are in your text. Since we are doing this "lecture" by long distance, we will not go into the particulars.
We shall use a little demonstration to show the usage and value of the *Heston Identity* and two of the direct results of the *Heston Identity*.

Question: Is cosine distributive? that is... Does: $\cos(x_2 - x_1) = \cos x_2 - \cos x_1$?

We consider $x_2 = \frac{\pi}{2}$ and $x_1 = \frac{\pi}{2}$ so we get:

$$\begin{aligned} \cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) &= \cos\frac{\pi}{2} - \cos\frac{\pi}{2} \\ \cos(0) &= \cos\frac{\pi}{2} - \cos\frac{\pi}{2} \\ 1 &= 0 - 0 \end{aligned}$$

Clearly $1 \neq 0$ so cosine (and no other trig function for that matter) is distributive. That leads us to seek out what

$\cos(x_2 - x_1) = \cos x_2 - \cos x_1$ is equal to. ....

To make a long story shorter ....

We have two important formulas

The "Cosine Add/Subtract" : $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

I read it as.... cosine of A plus or minus B equals cosine of the first, cosine of the second *disagree* sine of the first sine of the second. The *disagree* refers to the sign. If we had $\cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$ we would use the formula like this:

$$\begin{aligned} \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) &= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4} \\ &= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} \\ \cos\left(\frac{11\pi}{12}\right) &= \frac{-\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

The second formula is "The Sine Add/Subtract" formula: $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$$\begin{aligned} \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right) &= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{2\pi}{3} \\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\left(\frac{-1}{2}\right) \\ \sin\left(\frac{11\pi}{12}\right) &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

This formula is read: sine of A plus or minus B is sine of the first, cosine of the second *agree* sine of the second, cosine of the first.

Now, try using your calculator.... $\left(\frac{-\sqrt{2} - \sqrt{6}}{4}\right)^2 + \left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)^2 = \ ?$