We Don't Need No Stinkin' Pg 66

Open your text to page 66........ a wealth of information!. The first question that comes to mind is, "Why is this page *still* in the text book? .... Such an encyclopedia of information begs to be torn from the book and hung in a student's bedroom so he/she can see it upon awakening in the morning, just before drifting off to sleep at night and other inspirational times of the day.

But wait ....

Let's look closer ......

Section A .... obviously!

Section B .... $S^2 + C^2 = 1$ then divide by S^2 to get $1 + \cot^2 x = \csc^2$ x

or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

We *already* know the Heston Identity so this section is obvious!

Section C .... If I wanted to know what $\cos\left(\frac{\pi}{2} - x\right)$ was ....

I'd use the cosine add/subtract formula I already know! Obvious!

Section D .... Obvious. Cosine and secant are even ... they "absorb the negative", the other trig functions are called odd functions.

Section E .... This looks impressive but our version is more readable. We will use *our* version. Obviously!

Section F .... See the comment for section C. Obvious.

Section G .... Why memorize a zillion formulas? We know $2x = x + x$ and use the appropriate add identity. Obvious.

Section H .... Useful ... on occasion. Think .... $\frac{x}{2} + \frac{x}{2} = x$ and you can generate these in a flash. Almost obvious.

So.... the conclusion is.....

We know $\sin^2 x + \cos^2 x = 1$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$ and

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ and generally, that is all we really need to know.

What we really need is practice using these identities and we have plenty of that coming up right now...

The textbook offers two versions of "practice". **Verify** and **Prove** are the two types. The main difference between them is that when you **verify** an identity, you are allowed to manipulate both sides of the identity and in the case of a **Prove**, you are (generally) not allowed to alter one side or the other.

To make it less confusing, we *never verify*. We **always prove**. There are two distinct versions of **Prove**.

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Style 1 of **Prove** is to leave one side untouched ... make the other side absolutely identical to the untouched side using standard algebraic steps. This method is usually the easiest approach.
Style 2 of **Prove** is to begin with a known identity such as Heston's Identity and to manipulate both sides until the entire line ends up identical to the original problem.
One side-note is that it seems that the objective of the "problem writers" was to discover "similar but different" statements which don't look obvious. Some of us think they had too much time on their hands. In any case, they do supply us with problems for us to practice our knowledge of the basic trig formulas.

WTP stands for "Want To Prove". This tells you the objective of the problem.

Example of style 1:

WTP: $$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$$

$$= \frac{\cos x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}$$

$$= \frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{\cos x(1-\sin x)}{\cos^2 x}$$

$$= \frac{1-\sin x}{\cos x}$$

Example of style 2:

WTP: $$\sin x + \cos x \cot x = \csc x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{1}{\sin x} \cdot \left(\sin^2 x + \cos^2 x\right) = 1 \cdot \frac{1}{\sin x}$$

$$\sin x + \frac{\cos^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\sin x + \cos x \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x}$$

$$\sin x + \cos x \cdot \cot x = \csc x$$

Your success with these problems depends more on your algebraic skills than it does on your trigonometric skills. Don't skip steps. Perform organized, orderly, legible algebraic operations. There is no doubt that your trig skills will improve. We hope your algebraic skills will be reinforced so your confidence level will be near "mastery" level. Do *honors level* work!

One final note...

The text refers to "restrictions" when they show proofs and verifies. We will ignore all references to "restrictions" and any references to "properties". We cover this material in greater depth in Chapter 4. For now, just concentrate on proving the given identities.