H Trig

Day 5 notes

Mr. Mumaugh

We have a sine add/subtract formula $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$
and we have the cosine add/subtract formula: $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

so you know there has to be a tangent add/subtract formula. The bad news is that there are secant, cosecant and cotangent add/subtract formulas too. The good news is that we don't bother with them but we do know and love the tangent add/subtract formula:

$$\tan(A \pm B) = \frac{\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A}{\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B}$$

$$= \frac{\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A}{\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B} \infty \frac{\frac{1}{\cos A \cos B}}{\frac{1}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A \cos B}{\cos A \cos B} \pm \frac{\sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} \mp \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A}{\cos A} \pm \frac{\sin B}{\cos B}}{1 \mp \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Is the tangent an odd, an even (or neither) function?

$$\tan(0 - x) = \frac{\tan 0 - \tan x}{1 - \tan 0 \tan x}$$

$$= \frac{0 - \tan x}{1}$$

$$= -\tan x$$

so tangent is an odd function.

Tangent has period ≠. We saw that yesterday.... but with the tangent add/subtract formula it is much cleaner...

$$\tan(x + \neq) = \frac{\tan x + \tan \neq}{1 - \tan x \tan \neq}$$

$$= \frac{\tan x + 0}{1 - \tan x(0)}$$

$$= \frac{\tan x}{1}$$

$$= \tan x$$

A couple of fun examples on the next page.....

Given: $\tan x = 3 \qquad \neq < x < \frac{3\neq}{2}$

Find $\sin 2x$

$$\frac{\sin x}{\cos x} = 3$$
$$\sin x = 3\cos x$$
$$\sin^2 x = 9\cos^2 x$$
$$\sin^2 x = 9(1 - \sin^2 x)$$
$$\sin^2 x = 9 - 9\sin^2 x$$
$$10\sin^2 x = 9$$
$$\sin^2 x = \frac{9}{10}$$
$$\sin x = \pm\frac{3}{\sqrt{10}} = \pm\frac{3\sqrt{10}}{10}$$

$S^2 + C^2 = 1$ so $C^2 = 1 - S^2$.

but x is in the third quadrant so we want $-\frac{3\sqrt{10}}{10}$

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furthermore, since the Heston Identity says

$S^2 + C^2 = 1$

so $\infty\frac{9}{10}\infty + \cos^2 x = 1$

$$\cos^2 x = \frac{1}{10}$$

$$\cos x = \pm\frac{1}{\sqrt{10}}$$ since we are in the 3rd quadrant, cosine is negative so we want

$$\cos x = -\frac{1}{\sqrt{10}}$$

finally we are ready to compute $\sin 2x$....
since $\sin 2x = \sin(x + x) = \sin x\cos x + \sin x\cos x = 2\sin x\cos x$
so....

$$2\cdot\frac{-3}{\sqrt{10}}\cdot\frac{-1}{\sqrt{10}}$$
$$\frac{6}{10} = \frac{3}{5}$$

and the final answer $= \frac{3}{5}$

convenient values ....

Let's find the x when we know that $\tan x = -\frac{\sqrt{3}}{3}$.

1) Let's consider the number $\frac{\sqrt{3}}{3}$. We know that $\frac{\sqrt{3}}{3}$ is the answer you'd get when you rationalized $\frac{1}{\sqrt{3}}$.

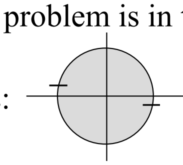
2) $\sqrt{3}$ ...... $\sqrt{3}$ ...... when you last saw a $\sqrt{3}$ in this class, what number was *under* it? 2? Yes that is $\frac{\sqrt{3}}{2}$.

3) $\frac{\sqrt{3}}{2}$ ...... $\frac{\sqrt{3}}{2}$ .....when you last saw a $\frac{\sqrt{3}}{2}$ in this class, what number was *with* it? $\frac{1}{2}$?

Yes that is, $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$ are "companions" aren't they?

4) Tangent is negative in the 2nd and 4th quadrants so that means x in this problem is in the 2nd or 4th quads.

5) so we have $-\frac{\sqrt{3}}{3}$ which is $-\frac{1}{\sqrt{3}}$ which is $-\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ which is $\frac{\sin x}{\cos x}$ thus:



$x \;\infty \frac{\infty 5\neq}{\infty 6}$, $\frac{11\neq}{6}\infty$