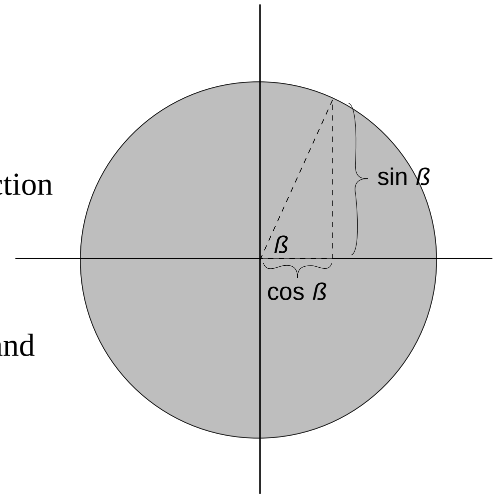
Day 4 Notes



$A(x) = \cos x$

D_x x ε R because we can go "around the circle in either direction as much or as little as we like.
ie. No restrictions.

R_A $-1 \le \cos x \le 1$. The least cosine can be is -1 (when $x = \pi$) and the most cosine can be is + 1 when $x = 0$.

Period 2π.

To prove any function has period "a", you *must* show that $f(x+a) = f(x)$.

example:
$$\begin{aligned}
\cos(x+2\pi) &= \cos x \cos 2\pi - \sin x \sin 2\pi \\
&= \cos x\ (1) - \sin x\ \ (0) \\
\cos(x+2\pi) &= \cos x
\end{aligned}$$

Thus, cosine has period 2π. This is reasonable because 2π represents a full revolution around the unit circle so one revolution from point X puts us back at the same spot.
Cosine is an even function.
A function is called even if $f(-x) = f(x)$. An even function "absorbs the negative sign." If you looked at the graph of an even function, you would see that the graph on the right side of the origin would look like a reflection of the graph on the left side of the origin.

example:
$$\begin{aligned}
\cos(0-x) &= \cos 0 \cos x + \sin 0 \sin x \\
&= (1)\cos x\ - (0)\sin x \\
\cos(-x) &= \cos x
\end{aligned}$$

so cosine is an even function.

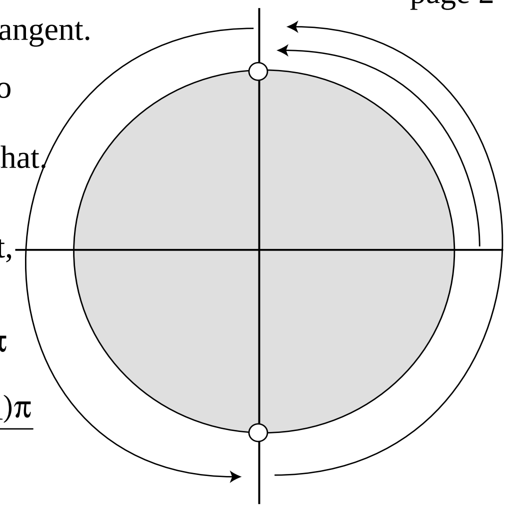
$B(x) = \sin x$

D_B x ε R for the same reason that cosine has a domain of all real numbers.
R_B $-1 \le \sin x \le 1$. See the picture listed with cosine. Sine is simply the vertical distance from the x-axis to the point in question.
Period 2π
odd function. A function is said to be an odd function if $f(-x) = -f(x)$. In an odd function, we can "factor out" the negative sign. The graph of an "odd function" reflects through the origin.

$$\begin{aligned}
\sin(0-x) &= \sin 0 \cos x - \sin x \cos 0 \\
&= 0 \cos x - \sin x\ (-1) \\
\sin(-x) &= -\sin x
\end{aligned}$$

so sine is an odd function.

$C(x) = \tan x$ which is defined to be the fraction: $\dfrac{\sin x}{\cos x}$ is known as Tangent.



Clearly, we cannot allow cosine to equal zero since division by zero is undefined. $\cos x = 0$ when $x = \dfrac{\pi}{2}$ and then every π length after that.

We start at zero.... We go $\dfrac{\pi}{2}$ to the first spot then π to the next spot, then π to the following spot, etc. We abbreviate this as $\dfrac{\pi}{2} + k\pi$

$$\frac{(2k+1)\pi}{2}$$

$D_C \quad x \not\in \left\{\dfrac{(2k+1)\pi}{2}\right\}$ and $R_D \quad \tan x \;\varepsilon\; R$

consider the fraction $\dfrac{\sin x}{\cos x}$. Let's consider the first quadrant so we are looking at positive values. The numerator has a maximum of one and a minimum of zero. The denominator has a maximum of one. The important item is that the denominator gets *smaller* thus you will be dividing by a smaller number making the quotient larger. Of course the smallest value the fraction can be is zero (when the numerator is zero and the denominator is 1). So, from zero, the value of tangent gets larger and larger without bound. In the second quadrant, the tangent is negative because sine is positive but cosine is negative (making the fraction negative). The magnitude can get large but its value will be negative. The conclusion is that the range of tangent is that $R_C \quad \tan x \;\varepsilon\; R$.

The period of tangent is not 2π like it is for sine and cosine. Remember, to prove a function has period "a", you must show that $f(x+a) = f(x)$.

$$\begin{aligned}
\tan(x+\pi) &= \frac{\sin(x+\pi)}{\cos(x+\pi)} \\
&= \frac{\sin x \cos\pi + \sin\pi \cos x}{\cos x \cos\pi - \sin x \sin\pi} \\
&= \frac{\sin x(-1) + 0\cos x}{\cos x(-1) - \sin x(0)} \\
&= \frac{-\sin x + 0}{-\cos x - 0} \\
&= \frac{\sin x}{\cos x} \\
\tan(x+\pi) &= \tan x
\end{aligned}$$

The “co” functions

$D(x) = \sec x$ which is called secant and is equal to $\dfrac{1}{\cos x}$.

Notice this function has the same situation as tangent, that is, the cosine is on the bottom therefore the secant function has the same domain restrictions as tangent. $D_D \quad x \notin \quad \left\{\dfrac{(2k+1)\pi}{2}\right\}$

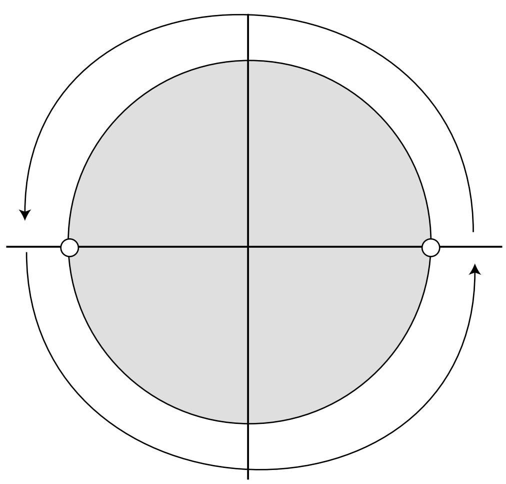
The range of secant has an important difference from tangent. The numerator is one. A fraction is smallest when its denominator is at its largest and the largest cosine can be is 1 so $\dfrac{1}{1} = 1$ is the smallest (positive) value attainable. Of course, secant can be negative. In that case the largest negative value which can be attained

is – 1.. Thus, the range of secant is given by: $R_D \quad \sec x \le -1$ or $\sec x \ge +1$

The period is 2π as is cosine and also like cosine, secant is an even function.

$E(x) = \csc x = \dfrac{1}{\sin x}$ and is known as cosecant.

The domain restrictions are computed in a similar fashion but with different results than they were computed for tangent and secant. We start at the first zero which is *x* = 0 and then travel π distance to the next (and all other zeros of the function).



so $0 + k\pi$

$k\pi$

Thus, $D_E \quad x \notin \quad \{k\pi\}$

$R_E \quad \csc x \le -1$ or $cscx \ge +1$ by the same reasoning as for the secant function.

period is 2π

Odd function.

$F(x) = \cot x = \dfrac{\cos x}{\sin x}$ known as cotangent. $D_F \quad x \notin \quad \{k\pi\}$ because sine is in the denominator like csc *x*.

$R_F \quad \cot x \;\; \varepsilon \;\; R$ for the same reasons as for tan *x*. Period is π and cotangent is an odd function.

$a(x)=\cos x$	$D_a \quad x \ \varepsilon \ R$	$R_a \quad -1 \le \cos x \le 1$	2π	even
$b(x)=\sin x$	$D_b \quad x \ \varepsilon \ R$	$-1 \le \sin x \le 1$	2π	odd
$c(x)=\tan x$	$D_C \quad x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$	$R_D \quad \tan x \ \varepsilon \ R$	π	odd
$d(x)=\sec x$	$D_d \quad x \notin \left\{\frac{(2k+1)\pi}{2}\right\}$	$R_d \quad \sec x \le -1$ or $\sec x \ge +1$	2π	even
$e(x)=\csc x$	$D_e \quad x \notin \{k\pi\}$	$R_e \quad \csc x \le -1$ or $\csc x \ge +1$	2π	odd
$f(x)=\cot x$	$D_f \quad x \notin \{k\pi\}$	$R_D \quad \cot x \ \varepsilon \ R$	π	odd

An example of what is meant by "evaluate":

Since tangent has period π, we can throw out any integer multiples of the angle. In this case, we toss out the 8π which is just 4 revolutions anyway and we are still at the same spot.

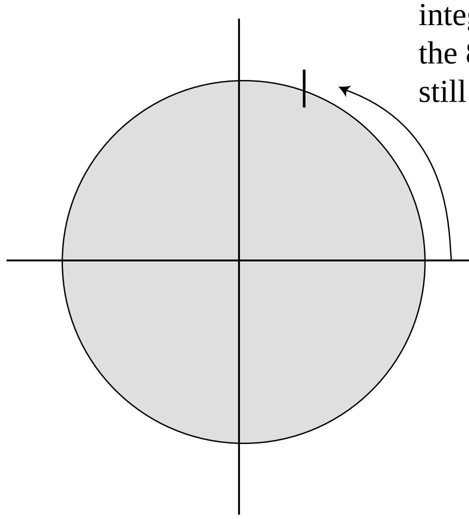
Evaluate $\tan\frac{25\pi}{3}$

$$\tan 8\frac{1}{3}\pi$$

$$\tan\frac{\pi}{3}$$

$$\frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$



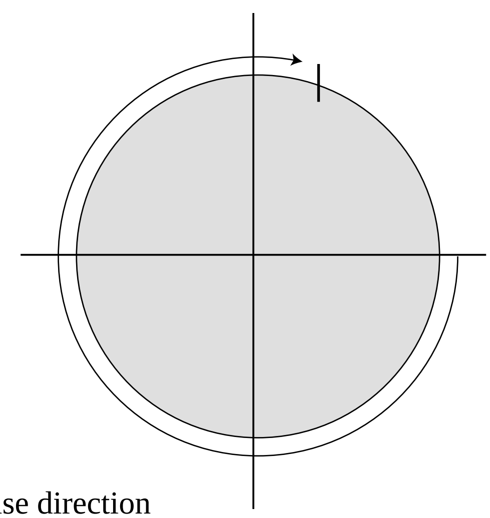
Evaluate $\sec\left(-\frac{5\pi}{3}\right)$

$$=\sec\frac{\pi}{3}$$

$$=\frac{1}{\cos\frac{\pi}{3}}$$

$$=\frac{1}{\frac{1}{2}}$$

$$=2$$



Notice that we measured in a clockwise direction because the arc measurement was negative.