Honors Analysis

Name ______________________________

Copy original problem. Per ________ Date ______________

Convince *me* that **you** understand the concept***!***

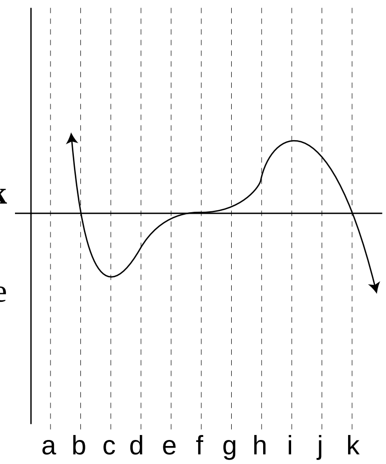
No Calculators.

Chapter 3 Exam

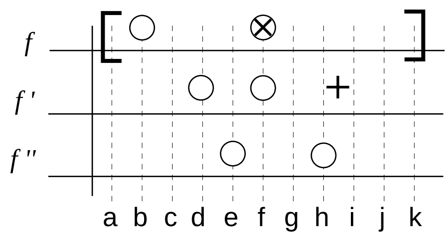
I Given the graph on the right: (20 pts tot)

a) Assume the given graph is $f(x)$.
Properly fill in the $f(x)$, $f'(x)$, $f''(x)$ ***numberlines on the back of this page.***.

b) Assume the given graph is $f'(x)$. **On the axis on the back of this paper** sketch $f(x)$. Specifically state coordinates of all extrema and PI. (Typically you might have a response such as: "$(a, f(a))$ f' test".) Justify, of course.



II Given: (30 pts tot)



a) Use the single y–axis and three x–axes **on the back of this page** to graph f, f', and f''.

b) Specifically state the coordinates of all extrema and points of inflection on f. Justify, of course. (As in Ib, above. Be sure to properly state coordinates.)

c) Specifically identify extrema on $f'(x)$. Justify. (Notice this asks about extrema in $f'(x)$)

III Given $f(x) = \dfrac{x}{x-1}$ for $-3 \le x \le 3$. (25 pts tot)

a) Using the definition of the derivative which yields a function, find $f'(x)$.

b) Using your answer to part a, find $f'\left(\frac{3}{2}\right)$

IV Given $f(x) = \begin{cases} |x+2| & if\ -\frac{5}{2} \le x < -1 \\ 2 - x^2 & if\ -1 \le x < 1 \\ 3 - 2x & if\ \ 1 \le x \le \frac{5}{2} \end{cases}$ (25 pts tot)

a) **Prove** f is continuous at $x = -1$

b) **Prove** f is continuous at $x = 1$

c) **Prove** $f'(-1)$ does or does not exist.

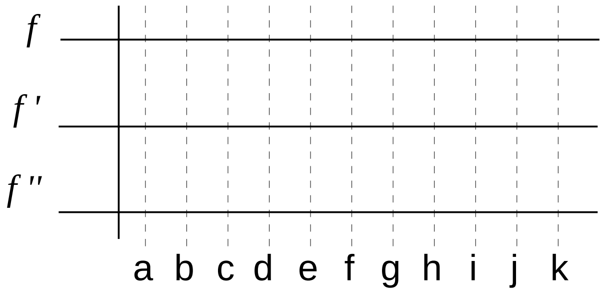
b) **Prove** $f'(1)$ does or does not exist.

e) **Prove** $f'(-2)$ does or does not exist.

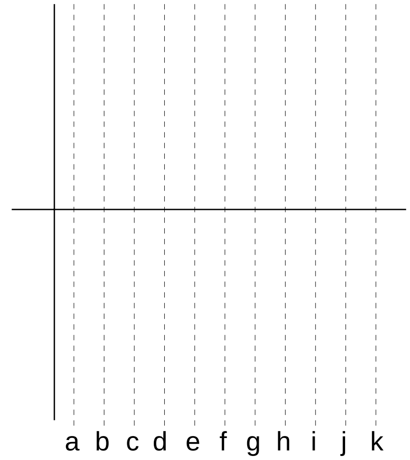
Extra Credit ------------------------------------ 5 pts ---

Draw the graph of $f(x)$ from section IV above.

I a



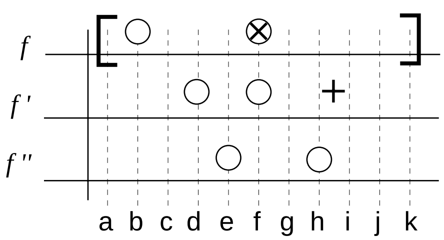
I b

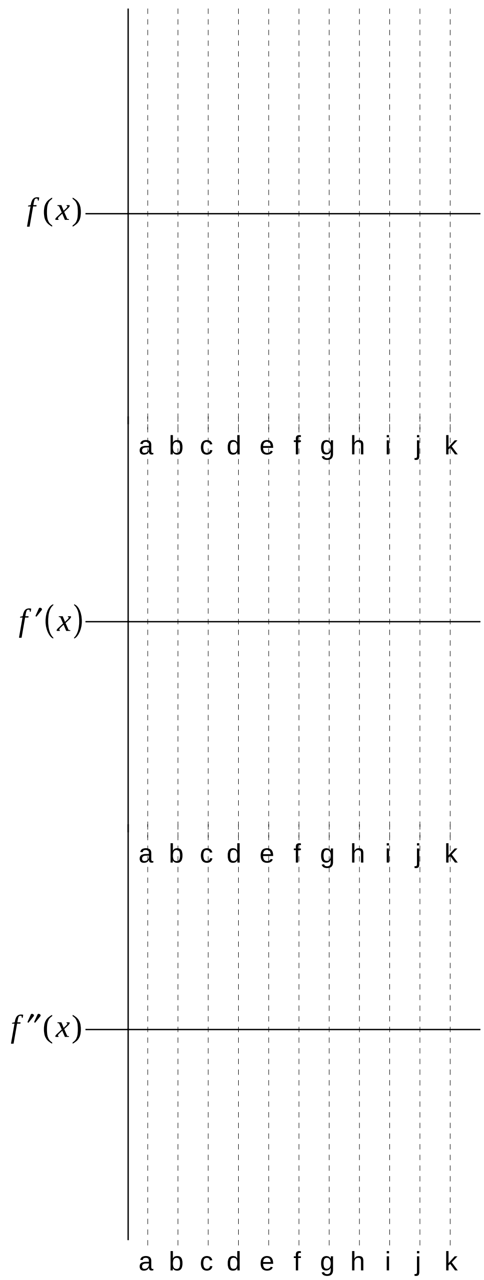


II

Below is the same graphic which is on the other side of this paper. It is reproduced here for your convenience. You are to fill in the graphic with pluses and minuses, etc. and then use the number lines to sketch the graph of the appropriate function on the proper axis.

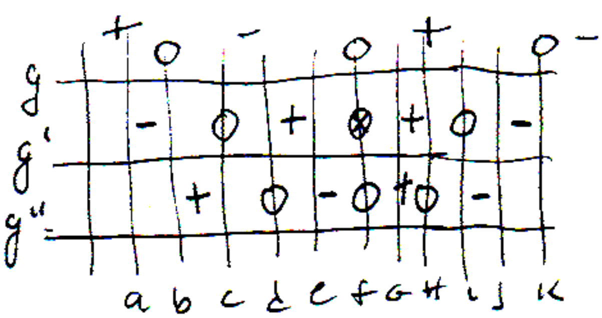
Put the rest of your answers on the newsprint as usual.



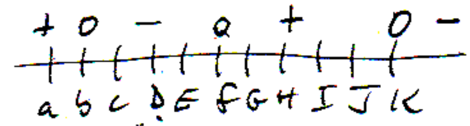


I

a)



b) $f'(x)$

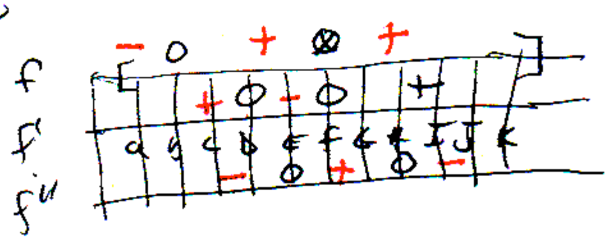


$(b, f(b))$ max f' test
$(f, f(F))$ min f' test
$(K, f(K))$ max f' test

PI when f' changes from increasing to Decreasing or vice versa.

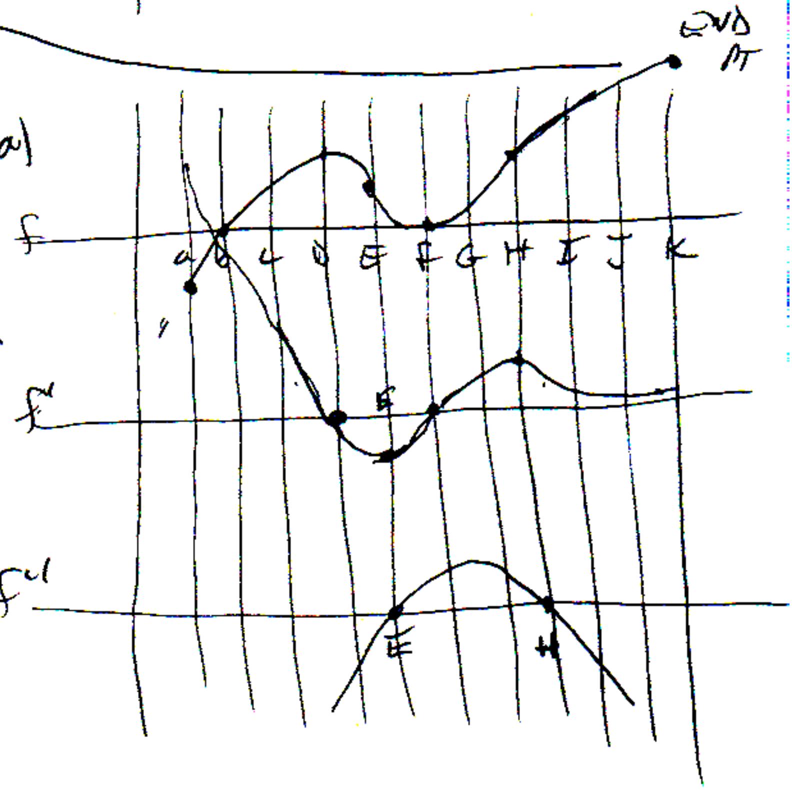
PI: $(C, f(c))$ $-$ to $+$ $(I, f(i))$ $+$ to $-$

II



min at f, $f(F)$ f' test. so $f'' > 0$
max at b, $f(b)$ f' test.
min at a, $f(a)$ ENDPT inc fn't.
max at K, $f(K)$ ENDPT inc to it.

a)



f' has: max $(a, f'(a))$ ENDPT dec fn't.
min $(e, f'(e))$ f' test.
max $(h, f'(h))$ f' test
min $(K, f'(K))$ ENDPT dec to it.

III

$$f(x) = \frac{x}{x-1}$$

a)
$$\lim_{h\to 0} \frac{1}{h}\left(\frac{x+h}{x+h-1} - \frac{x}{x-1}\right)$$
$$\lim_{h\to 0} \frac{1}{h}\left[\frac{x^2+xh-x-h-x^2-xh+x}{(x-1)(x+h-1)}\right]$$
$$\lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{(x-1)(x+h-1)}\right]$$
$$\frac{-1}{(x-1)^2}$$

b)
$$f'\left(\tfrac{3}{2}\right) = \frac{-1}{\left(\frac{3}{2}-1\right)^2}$$
$$\frac{-1}{\left(\frac{1}{2}\right)^2}$$
$$\frac{-1}{\frac{1}{4}}$$
$$-4$$

IV

$$f(x) = \begin{cases} -x-2 & \text{if } -\frac{5}{2} \le x < -2 \\ x+2 & \text{if } -2 \le x < -1 \\ 2-x^2 & \text{if } -1 \le x < 1 \\ 3-2x & \text{if } 1 \le x \le \frac{5}{2} \end{cases}$$

Not a Proof −C
one sided limits only −3

a) $\lim_{x\to -1} f(x) = f(-1)$

$f(-1) = 1$ ①

$\lim_{x\to -1^-} x+2 = -1+2 = 1$ $\lim_{x\to -1^+} 2-x^2 = 2-1 = 1$

$=$ ② $=$ ③

∴ f cont at $x=-1$

b) $f(1) = 3(1)-2 = 1$ ①

$\lim_{x\to 1^-} 2-x^2 = 2-1 = 1$ $\lim_{x\to 1^+} 3x-2 = 3-2 = 1$

$=$ ② $=$ ③

∴ f cont at $x=1$

c) $\lim_{x\to -1} \frac{f(x)-1}{x+1}$

$\lim_{x\to -1^-} \frac{x+2-1}{x+1}$ $\lim_{x\to -1^+} \frac{2-x^2-1}{x+1}$

$\lim_{x\to -1^-} \frac{x+1}{x+1}$ $\lim_{x\to -1^+} \frac{1-x^2}{x+1}$

$\lim_{x\to -1^-} 1$ $\lim_{x\to -1^+} \frac{(1-x)\cancel{(1+x)}}{\cancel{x+1}}$

1 $\lim_{x\to -1} 1-x$

$1 \ne 2$ (L) ≠ (R)

∴ limit DNE so $f'(-1)$ DNE

D) $\lim_{x\to 1} \frac{f(x)-1}{x-1}$

$\lim_{x\to 1^-} \frac{2-x^2-1}{x-1}$ $\lim_{x\to 1^+} \frac{3-2x-1}{x-1}$

$\lim_{x\to 1^-} \frac{1-x^2}{x-1}$ $\lim_{x\to 1^+} \frac{2-2x}{x-1}$

$\lim_{x\to 1^-} \frac{\cancel{(1-x)}(1+x)}{\cancel{(x-1)}(-1)}$ $\lim_{x\to 1^+} \frac{-2(x-1)}{x-1}$

$-2 = -2$

∴ $f'(1)$ exists and is -2

E) $\lim_{x\to -2^-} \frac{-(x+2)-0}{x+2}$ $\lim_{x\to -2^+} \frac{x+2-0}{x+2}$

$\lim_{x\to -2^-} \frac{-(x+2)}{x+2}$ $\lim_{x\to -2^+} \frac{x+2}{x+2}$

$-1 \ne 1$

limit DNE ∴ $f'(-2)$ DNE

(L) ≠ (R)

Extra.