Advanced Placement Calculus

Name _____

Copy original problem.

Per _____   Date _____

Convince *me* that **you** understand the concept*!*

No calculators!   **First Exam**

I   Write as a piece-wise function then sketch.   (10 pts ea)

a) $f(x) = \left| x + \left| x + 1 \right| \right|$

b) $g(x) = \left| x \right| + \left| x - 1 \right|$

II   Solve for all $x \; \varepsilon \; \Re$. Graph solution on a number line.   (10 pts ea)

a) $\dfrac{\left| 9 - 2x \right|}{x - 5} \le \dfrac{7}{x - 5}$

b) $\left| x^2 \right| < \left| -5 \right|$

III   Given: $f(x) = \dfrac{x^2 + 2x}{x^2 + x - 2}$ (15 pts tot)

a)   Solve $\forall \; x \, \varepsilon \, \Re$: $f(f(x)) = x$

b)   Using the definition of the derivative ($h \to 0$), determine $f'(x)$.

IV   Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not.   (5 pts ea)

a) $\displaystyle\lim_{x \to -\infty} \dfrac{\sqrt{x^2 - 4}}{3x + 4}$

b) $\displaystyle\lim_{x \to \infty} \left(\dfrac{x^2 - \sqrt{x}}{x^2 + x\sqrt{x}} \right)$

c) $\displaystyle\lim_{x \to \infty} \sqrt{x^2 + x} - \sqrt{x^2 + 4}$

d) $\displaystyle\lim_{x \to 0} \left[\dfrac{3 + 2x}{x + 5x^2} - \dfrac{3}{x} \right]$

e) $\displaystyle\lim_{x \to -2} \dfrac{\sqrt{x^2} - 2}{x + 2}$

f) $\displaystyle\lim_{x \to 4^+} \left(\dfrac{1}{x} - \dfrac{1}{4} \right)$

g) $\displaystyle\lim_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4} \right) \left(\dfrac{1}{x - 4} \right) \right]$

h) $\displaystyle\lim_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4} \right) (x - 2) \right]$

i) $\displaystyle\lim_{x \to 4^+} \left[\left(\dfrac{1}{x} - \dfrac{1}{4} \right) \left(\dfrac{1}{x - 4} \right)^2 \right]$

Extra Credit ---------------------------------- **5 pts** --

Jake can run around a circular track in forty seconds. Jed, running in the opposite direction, meets Jake every fifteen seconds. How long does it take Jed to run around the entire track, expressed in seconds?