Advanced Placement Calculus

Name ________________________________

Copy original problem.

Per ________ Date ______________

Convince *me* that **you** understand the concept***!***

Assume constants of integration to be zero.

Chapter 9 Exam

I *Given*: $f(x)=\sqrt{\dfrac{1+x}{1-x}}$. This problem concerns $\int f(x)\ dx$. (70 pts tot)

a) Let $z^2=\dfrac{1+x}{1-x}$. Show that the $\int f(x)\ dx$ can be rewritten as $\displaystyle\int \frac{4z^2}{\left(z^2+1\right)^2}\ dz$

b) Make a trig substitution in $\displaystyle\int \frac{4z^2}{\left(z^2+1\right)^2}\ dz$ and then integrate.

Leave your antiderivative in terms of θ.

c) Let $x=\cos 2t$. Rewrite dx so that it involves only single angle arguments (ie t rather than $2t$).

Substitute into $\displaystyle\int\sqrt{\frac{1+x}{1-x}}\,dx$. Integrate. Leave your antiderivative in terms of t.

d) Multiply $\displaystyle\int\sqrt{\frac{1+x}{1-x}}\,dx$ by the fraction : $\dfrac{\sqrt{1+x}}{\sqrt{1+x}}$. Integrate the result.

e) Sketch the graph of $\displaystyle\int\sqrt{\frac{1+x}{1-x}}\,dx$ (from your grapher) using window $x\ \varepsilon\ [-2,2]$ and $y\ \varepsilon\ [0,5]$.

f) Rewrite $\displaystyle\int\sqrt{\frac{1+x}{1-x}}\,dx$ with limits of integration $x=-1$ to $x=+1$. What do you notice that might be significant? Considering the substitution made in part "a", the new limits would be from $z=0$ to $z=\infty$. In part " b " you made a trig substitution. Compute the new limits of integration in terms of θ and then evaluate your antiderivative answer to part "b".

g) Consider the changes made in part "c" above including the introduction of limits to be from $x=-1$ to $x=+1$, compute the new limits of integrations in terms of t and then evaluate your antiderivative from part " c ".

h) Consider the changes made in part "d" above including the introduction of limits to be from $x=-1$ to $x=q$. Compute the new limits of integration and then evaluate your antiderivative from part " d ". Apply the limit as $q\to 1^-$.

i) Use your calculator to compute FnInt($\sqrt{\dfrac{1+x}{1-x}}$, x, – 1, 1)

II Integrate: $\displaystyle\int\frac{7x^2-4x}{\left(x^2+1\right)(x-2)}\,dx$ (No calculator) (15 pts)

III Use the method "parts" **only one time** and the fact that $\sin^2\theta=\sin\theta\cdot\sin\theta$ to integrate: $\int\sin^2\theta\,d\theta$

(15 pts)

Extra Credit ----------------------------------- 5 pts --

Use the method of partial fractions to rewrite the revised integral for part I a. Do not integrate.