A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!

Name ______________________________

Per __________ Date ____________

Chapters 7 & 8 Exam

I Use the *definition of* ***definite integral*** to show that $\int_2^4 6\ dx = 12$.

(hint: The *definition* has nothing to do with a graph.) (10 pts)

II Differentiate: $g(x) = \int_{x^2}^{2} \sqrt{\cos t}\ dt + \int_2^3 \sqrt{\cos t}\ dt$ (10 pts)

III Given the three equations: $x^2 + y^2 = 4$, $y = x + 2$, and $y = x$. (20 pts total)

A) Sketch the three equations on the same axis. Shade the region above the x-axis bounded by these curves.

B) Set up, but **Do Not Evaluate** the definite integral(s) which describe the shaded region in part A. Give a very explicit explanation of your methods.

IV The value $\frac{1}{b-a}\int_a^b f(x)\ dx$ is called the mean (or average) value of f on the interval [a, b] and is usually denoted f_{av}. Let $f(x) = \sin x^2$ for $0 \le x \le \sqrt{\pi}$ (20 pts total)

a) Describe the window setting necessary to yield a clear view of the function in the given interval on your calculator.

b) Sketch your graph.

c) Predict the mean value, f_{av}, of f on $[0, \sqrt{\pi}]$. Call your prediction *A*.

d) Use the program **RiemannC** with $n = 100$ to find the approximation of the integral. Compute f_{av} using this value and call this value *B*.

e) Use **fnInt** $(Y_1, X, 0, \sqrt{\pi})$ / $\sqrt{\pi}$ (found in the MATH menu). Call this value *C*.

f) Compare the values *A*, *B*, and *C*. Briefly explain.

V The velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown in a table of values for $v(t)$, at 5 second intervals of time.

Approximate the definite integral, $\int_0^{50} v(t)dt$ with a riemann sum, using the mid-points of five subintervals of equal length. Using correct units, explain the meaning of this integral. (20 pts)

t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

VI Given $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x}$. Draw the graph showing the area computed and compute the exact area described. (tot 20)

a) $\int_1^2 f(x)\ dx - \int_1^2 g(x)\ dx$

b) $\int_0^1 f(x)\ dx + \int_1^2 g(x)\ dx$

Extra Credit **5 pts**

Lemma 3, used by our text in the proof of the Fundamental Theorem of Calculus states:

$f(X)(b-a) = \int_a^b f(x)\ dx$. Using the function in Section **IV**. Compute the "cap *X*" value.