Advanced Placement Calculus Name _____

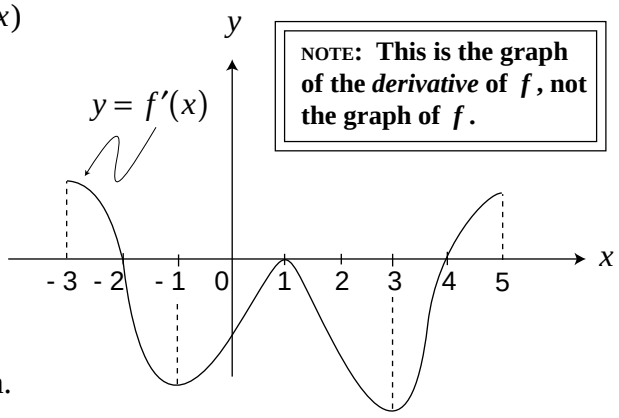
Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

Chapter 6 Mechanical Exam

I

The figure shows the graph of $f'(x)$, the derivative of a function $f(x)$. The domain of $f(x)$ is the set of all real numbers x such that $-3 \le x \le 5$. (40 pts tot)



NOTE: **This is the graph of the *derivative* of f, not the graph of f.**

a) For what values of x does f have a relative maximum? Explain.

b) For what values of x does f have a relative minimum? Explain.

c) On what intervals is the graph of f concave upward? Use $f'(x)$ in your justification.

d) Suppose that $f(1) = 0$. On an *xy*-plane, draw a sketch that shows the general shape of the graph of the function f on the open interval $0 < x < 2$.

II

This problems deals with functions defined by $f(x) = x + b\sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$. (40 pts tot)

a) Sketch the graphs of two of these functions: $y = x + \sin x$ and $y = x + 3\sin x$ on the same axis.

b) Find the *x*-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b\sin x$.

c) Are the points of tangency described in part (b) relative maximum points of f? Why?

d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

III

Find the following limits. (All exist). Use l'hospital's rule at least once.
Be sure your reasoning is very clear. (20 pts tot)

a) $\lim_{x \to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos 2x}$ b) $\lim_{x \uparrow \pi/2} (2x - \pi)\sec x$ c) $\lim_{x \downarrow 0} \left(\dfrac{1}{e^x - 1} - \dfrac{1}{x} \right)$

Extra Credit ---------- 5 pts ----------

Given: $f(x) = \dfrac{2}{5}(1+x)^{\frac{5}{2}} - \dfrac{2}{3}(1+x)^{\frac{3}{2}}$ and $g(x) = \dfrac{2x}{3}(1+x)^{\frac{3}{2}} - \dfrac{4}{15}(1+x)^{\frac{5}{2}}$.

Find $f'(x)$ and $g'(x)$. What do your answers suggest? What is that constant?