Advanced Placement Calculus Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

No calculators, of course.

Chapter 6 Mechanical Exam

I

Evaluate the following limits. Use l'hopital's rule at least one time.
(Be very sure your methods are explicit.): (50 pts tot)

a) $\lim_{x\to 0^+}\left[x^{\left(\frac{\ln a}{1+\ln x}\right)}\right]$ b) $\lim_{x\to\infty}\left[x^{\left(\frac{\ln a}{1+\ln x}\right)}\right]$ c) $\lim_{x\to 0}\left[(x+1)^{\left(\frac{\ln a}{x}\right)}\right]$

d) $\lim_{x\to\infty}\frac{\frac{\pi}{2}-\tan^{-1}x}{\frac{1}{x}}$ e) $\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\sin\frac{1}{x}}$ f) $\lim_{x\to 0}\frac{\tan^{-1}x}{\tan^{-1}2x}$

g) $\lim_{x\to 0}(1+\sin 4x)^{\cot x}$

II

A function is said to have a *fixed point* on an interval (*a*, *b*) if there is some point $x = p,\ a < p < b$ such that $f(p) = p$. Show that $f(x) = 4x^3 - 1$ has a fixed point on the interval $\left(\frac{1}{2},1\right)$. (10 pts)

III

Given $f(x) = \ln(2+\sin x)$ for $0 \le x \le 2\pi$. (25 pts tot)

a) Find and identify coordinates of all extrema.

b) Find coordinates of all points of inflection.

c) Sketch. Be sure all features are clearly labeled.

IV

Given $y = x^3 + ax^2 + bx + 1$ for $x\ \varepsilon\ [0,2]$. (15 pts tot)

a) Find the values of a and b so that (1, 6) is an inflection point.

b) Find and identify all extreme values.

Extra Credit ---------- 5 pts ----------

Find a and b so that: $\lim_{x\to 0}\frac{\cos(ax)-b}{2x^2} = -4$