

# Advanced Placement Calculus

Name \_\_\_\_\_

Copy original problem.

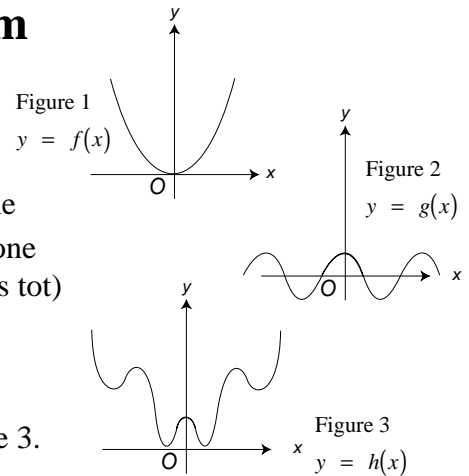
Per \_\_\_\_\_

Date \_\_\_\_\_

Convince *me* that **you** understand the concept!

## Chapter 6 Mechanical Exam

**I** Let  $f(x) = x^2$ ,  $g(x) = \cos x$ , and  $h(x) = x^2 + \cos x$ . From the graphs of  $f$  and  $g$  shown on the right in figure 1 and figure 2, one might think that the graph of  $h$  should look like figure 3. (15 pts tot)



- Sketch the actual graph with  $x \in [-6, 6]$  and  $y \in [\pm 6, 40]$
- Use  $h''(x)$  to explain why the graph of  $h$  does not look like figure 3.

**II** Given  $f(x) = x^{\frac{5}{3}} + 5x^{\frac{2}{3}}$  (30 pts tot)

- Draw and fill in  $f$ ,  $f'$ , and  $f''$  number lines.
- Determine the coordinates of and identify *all* significant points.
- Specifically state intervals where  $f$  is increasing, decreasing, concave up, and concave down.
- Sketch  $f(x)$

**III** Find the following limits (all limits exist) (5 pts ea)

a)  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$       b)  $\lim_{x \rightarrow 1} \frac{e^x - e}{5 \ln x}$       c)  $\lim_{x \rightarrow 0^+} (\sin x)^{\sin x}$       d)  $\lim_{x \rightarrow 0^+} x^{\frac{2}{\ln x}}$

e) Agree or disagree. Support your position. "Any number to the zero power is one."

**IV** Given  $f(x) = \ln x$  and  $g(x) = kx$ . Determine  $k$  so that  $g$  is tangent to  $f$ . (10 pts)

**V** Define each of the following. For each, draw a sketch illustrating your definition. Be sure to indicate all points mentioned. Do not use specific functions nor numbers. (20 pts tot)

- Rolle's Theorem
- Law of the Mean (Mean Value Theorem)
- Intermediate Value Theorem

**Extra Credit** ----- 5 pts -----

Let  $f(x) = x + \cos x + 3$  and  $g(x) = 3x^3 - 2x + 4$ .  
Determine the coordinates (accurate to 3 places) of the points of intersection of  $f$  and  $g$ .