A.P. Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators, of course.

Name _____

Per _____ Date _____

Chapter 6 Mechanical Exam

I Evaluate the following limits. Use l'Hopital's rule at least once, if possible. (tot 20 pts)

A) $\lim_{x \to 1} \frac{x-1}{\sqrt{x^2+3}-2}$ B) $\lim_{x \to e} \frac{e^x - x^e}{x^x - e^e}$ C) $\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$

II Given $f(x) = \frac{x^3 - x}{x^3 - 4x}$ (tot 30 pts)

A) Find $\lim_{x \to 0} f(x)$.

B) Find the zeros of *f*.

C) Write an equation for each vertical and each horizontal asymptote to the graph of *f*.

D) Describe the symmetry of the graph of *f*.

E) Sketch the graph of *f*. Be very sure you explain behavior of graph (use limits).

III Given $f(x) = 3x + 1 - x^2$. Prove that the graph of f is ***not above*** the graph of the tangent line to f at (1,3) for all *x*. (25 pts)

IV Let f be a function defined by $f(x) = \left(x^2 + 1\right)e^{-x}$ for $-4 \le x \le 4$. (tot 25 pts)

A) What are the coordinates of all extrema? Identify and justify, of course.

B) What are the coordinates of all points of inflection of *f*. Justify, as usual.

Extra Credit **5 pts**

Let f be the function defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

Using the definition of the derivative, prove that f is differentiable at $x = 0$.