Advanced Placement Calculus Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

No Calculators, as usual.

Chapter 6 Mechanical Exam

I Find the following limits. (All limits do exist.) Be very sure your reasoning is clear. (5 pts ea)

a) $\lim_{x \to 1} \dfrac{x^{1/3} - x^{1/4}}{x^{1/3} - x^{1/5}}$ b) $\lim_{x \to \infty} \dfrac{x^{1/3} - x^{1/4}}{x^{1/3} - x^{1/5}}$ c) $\lim_{x \to 1} x^{\frac{1}{\ln x}}$

II Given the real numbers a and $b > 0$ (40 pts tot)

a) Argue that: $0 < \dfrac{a}{a + b} < 1$

b) Given $f(x) = |x|$. Using the fact that $f(x) = \sqrt{x^2}$, show that $f'(x) = \dfrac{x}{|x|}$.

c) Given $g(x) = |x|^a \, |x - 1|^b$ and $g'(x) = |x|^{a-2} \, |x - 1|^{b-2} \, x(x - 1)(a + b)\left[x - \dfrac{a}{a + b}\right]$

1) For which x does g have a relative maximum?

2) Show that this relative maximum value is $\dfrac{a^a b^b}{(a + b)^{a+b}}$

III Given $f(x) = \dfrac{x}{(x - 1)^2}$. Determine extrema, state coordinates of points of inflection and draw the graph of the function showing these points. (20 pts)

IV For which $x > 0$ does $f(x) = x^x$ achieve its minimum value? (10 pts)

V Prove that $f(x) = \dfrac{\sin x}{x}$ is a decreasing function for $0 < x < \pi/2$ (15 pts)

Extra Credit ---------- 5 pts ----------

Show that the derivative of $g(x)$ in part II C is as it is stated.