## Advanced Placement Calculus

Copy original problem.
Convince me that you understand the concept!

## Chapter 6 Applications Exam

I
A poster is to contain fifty square inches of printer matter with margins of four inches at the top and bottom and two inches on each side. Find the overall dimensions of the poster if the total area of the poster is to be a minimum.

II
Given $f(x)=x^{2}+\frac{c}{x}$ where " $c$ " is a constant.
(30 pts tot)
a) Find $c$ such that $f$ has a relative minimum at $x=2$.
b) Find $c$ such that $f$ has a relative minimum at $x=-3$.
c) Find $c$ such that $f$ has a point of inflection at $x=1$.
d) Show that $f$ cannot have relative maximum for any value of $c$.

During a cough there is a decrease in the radius of the person's trachea (windpipe). Suppose that the normal radius of the trachea is $R$ centimeters and the radius of the trachea during a cough is $r$ centimeters, where $R$ is a constant and $r$ is a variable. The velocity of air through the trachea can be shown to be a function of $r$, and if $V(r)$ centimeters per second is this velocity, then $V(r)=k r^{2}(R-r)$ where $k$ is a positive constant and $r$ is in $\left[\frac{1}{2} R, R\right]$. Determine the radius of the trachea during a cough for which the velocity of air through the trachea is greatest.

Find the area of the largest isosceles triangle that can be drawn with one vertex at the origin and with the others above the vertex on a line parallel to the $x$-axis and on the parabola $y=27-x^{2}$.

( 25 pts )

## Extra Credit

On a two-mile race track, you bike the first mile at ten MPH. At what rate must you bike the second mile to average twenty MPH for the entire track?

