Advanced Placement Calculus

Name _____

Copy original problem.　　　　　　　　Per _____　　　　　Date _____

Convince *me* that **you** understand the concept*!*
Each section 25 points. All extrema problems require
proper justification. If decimals are used, they must be
accurate to 3 places right of the point.

Chapter 6 Applications Exam

I　A rectangular outdoor pen is to be added to an existing animal house.
See the figure on the right. If 85 meters of new fence are to be used,
what should be the dimensions of the new pen in order to maximize its
area. Of course, no fence will be needed along the walls common to
the new pen and the old animal house.

Existing House

10 m

5 m

dotted lines are
the new fence

New Pen
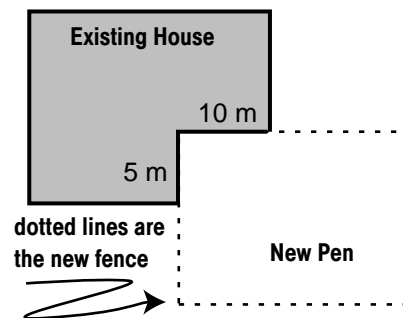
II　In a transatlantic cable, let x be the ratio of the radius of the core of
the cable to the thickness of the covering for the cable. The speed s of
a signal passing through the cable is given by:　$s = -kx^2 \ln x$　for $0 < x < 1$　where k is a positive
constant. What will maximize s ?

III　The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will
yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by
each tree decreases by 12 apples per year due to competition for water, nutrients, sunlight, etc. (Thus, for
example, if 25 trees were planted per acre, then *each* tree would produce $600 - 12(25 - 24) = 588$ apples
per year.)

　a)　If x is the number of trees per acre, write an expression for the number of apples which will produced
　　　by each tree (i.e.. apples per tree).

　b)　Continuing from (a), write a function for the number of apples *per acre*. (Remember, there are x trees
　　　per acre.)

　c)　Determine the number of trees per acre which will produce the most apples per acre. How many more
　　　apples will be produced at this level?

IV　At what first-quadrant point on the parabola $y = 4 - x^2$ does the tangent, together with the coordinate
axes, determine p so the triangle has minimum area (no decimals)? What is that area (decimals allowed
here)?　(Hint: let the tangent point have coordinates $(p, 4 - p^2)$)

Extra Credit ---------------------------------- **5 pts** ---

In one model of the spread of a contagious disease among members of a population of M people, the
incidence of the disease, $R,$ measured as the number of new cases per day is given in terms of the number,
x, of individuals already infected by: $R(x) = kx(M - x)$ where k is some positive constant. How many
individuals in the population are infected when the incidence, R is the highest?