Advanced Placement Calculus

Name _____

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept***!***

No decimal answers.

Chapter 5 Exam

I Determine if $x^3 + x^2 = \sqrt{x+2}$ is solvable. (20 pts)

a) Using a window with $x \in [-2, 2]$ $y \in [-1, 4]$
graph $y = x^3 + x^2$ and $y = \sqrt{x+2}$. Sketch your picture on your paper.

b) What conclusion can you make concerning the objective of this problem and your results in part (a)?

c) Define a function, $f(x)$ which is the difference of the two given equations. Find the narrowest interval $[a, b]$ such that a and b are integers and that $f(a)$ and $f(b)$ are integers which have different signs. Explain, of course.

d) Using the material above in part, prove the objective of this problem. (ie. Prove a solution exists.) Give a precise answer. Be very specific. Allow for no misinterpretations.

II Show that the derivative of : $\left(\sqrt{1+x}\right)\left(\sqrt{2-x}\right) - 3\sin^{-1}\sqrt{\frac{2-x}{3}}$ is $\frac{\sqrt{2-x}}{\sqrt{1+x}}$ (20 pts)

III Given: $f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

Find all three points on the graph of f whose tangent lines pass through (4, 3) (20 pts)

IV Given y. Find $\frac{dy}{dx}$ then evaluate $\frac{dy}{dx}$ at the indicated abscissa value. (10 pts ea)

a) $y = \ln\left|x + \sqrt{x^2 - 25}\right| + \ln\left|x - \sqrt{x^2 - 25}\right|$ ..... $x = \sqrt{5}$

b) $y = (\arctan x)^{\cos x}$ ..... $x = \frac{\pi}{4}$

c) $y = x^{x^x}$ ..... $x = 2$

d) $y = e^x + x^e$ ..... $x = \ln 2$

Extra Credit ---------- 5 pts ----------

Jane can clean the bamboo hut in seven hours and Tarzan can clean the hut in eight hours. Being accurate to the nearest minute, if they both start cleaning at 2 p.m., what time will they finish?