Advanced Placement Calculus

Name ______________________________

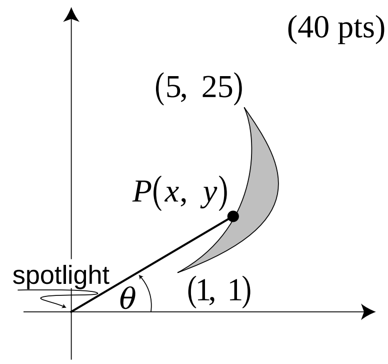
Copy original problem. Per ________ Date _____________

Convince *me* that **you** understand the concept***!***

Chapter 11 etc   Exam

I

The figure on the right shows a spotlight shining on point $P(x,y)$ on the shoreline of Crescent Island. The spotlight is located at the origin and is rotating. The portion of the shoreline on which the spotlight shines is in the shape of the parabola $y = x^2$ from the point (1, 1) to the point (5, 25). Let θ be the angle between the beam of light and the positive x-axis. (hint: soh-cah-toa). (40 pts)



a) For what values of θ between 0 and 90 degrees does the spotlight shine on the shoreline?

b) Explain how we know the (x,y) coordinates of point P in terms of θ are given by: $\left(\tan\theta, \tan^2\theta\right)$.

c) The spotlight is rotating at the rate of one revolution per minute. We can define "speed along a curve" as: $speed = \dfrac{ds}{dt} = \sqrt{(\dot{x})^2 + (\dot{y})^2}$ (Notice that *speed* is a change wrt *time*.). How fast is the point P traveling along the shoreline at the instant when it is at the point (3,9)? **Be exact. No decimals.**

II

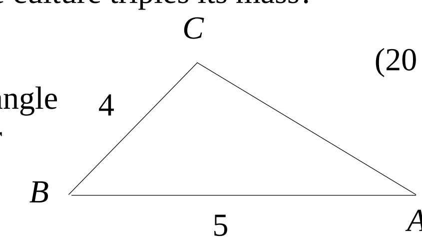
Related Rates. Answer using a complete, meaningful sentence. (Properly interpreted, of course.)

a) Suppose that a bacterial culture grows at a rate proportional to the amount present. At 2:30 P.M. there are ten grams of the culture and at 3:30 P.M. there are twenty grams.. (total 20 pts)

1) Derive the formula for the amount y of culture present in terms of time, t .

2) What time (to the nearest minute) will it be when the culture triples its mass?

b) Given triangle ABC as shown on the right. The angle B is increasing at .06 radians/second. How fast is the area of the triangle changing when angle B is 60 degrees? Is the area increasing or decreasing. Will it continue to increase (or decrease) or when will it change? (20 pts)



III

Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

Extra Credit ----------------------------------- 5 pts ---

A stone is dropped into a cylindrical water tank which is two feet in diameter causing the water level to rise one and one-half inches. What is the exact volume of the stone? $V_{cylinder} = \pi r^2 h$