Advanced Placement Calculus

Name ________________________________

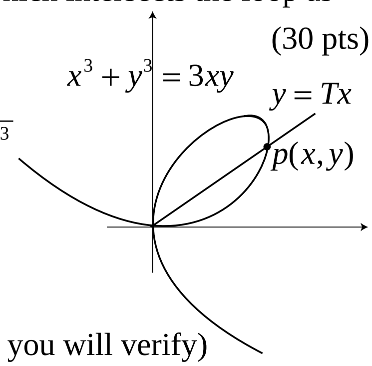
Copy original problem. Per ________ Date ______________

Convince *me* that **you** understand the concept***!***

Chapter 11 Exam

I

Given the graph of the folium of Decartes. A folium is a plane cubic curve having a single loop, a node, and two ends asymptotic to the same line. We have drawn the line $y = Tx$ which intersects the loop as shown at point P which has coordinates (x, y). (30 pts)



a) Write x in terms of T. Explain the steps necessary to arrive at $x = \dfrac{3T}{1+T^3}$

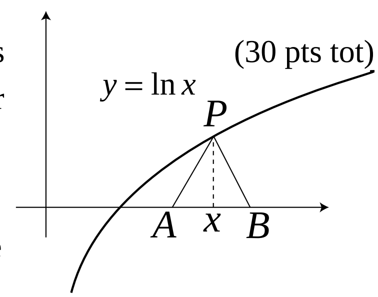
b) Write y in terms of T .

c) Find the equation of the tangent line when $T = 1$.

d) Find the coordinates of that point of the loop which is highest. (of course, you will verify)

II

A point P with coordinates (x, y) moves along the curve $y = \ln x$ so that its abscissa value (that is the x-coordinate) increases at the rate of $\sqrt{3}$ inches per second. P is one vertex of an equilateral triangle whose other two vertices on the x-axis. (hint: let the line AB have length $2k$.) (30 pts tot)



a) How fast is the area of triangle APB changing when the ordinate of P (the y-coordinate) value is one?

b) As x continues to get larger, what can be said about the rate at which the area of the triangle is growing?

c) How fast is the perimeter of triangle APB changing when the ordinate of P (the y-coordinate) value is one?

III

If the rate at which an ice cube melts (in cubic inches per second) is directly proportional to the total surface area of the cube with proportionality constant k, show that a single edge of the cube decreases at a constant rate and that the area of a side does ***not*** change at a constant rate. (20 pts)

IV

Describe the similarities and differences between the Cartesian and the polar graphing planes. Include in your discussion directions for converting from one form to the other and back again. List, identify and label with the Cartesian as well as the polar equations: one for a straight line through the pole with positive slope, one for a straight line through the pole with negative slope, one for a straight line with positive slope NOT through the pole; one for a straight line with negative slope NOT through the pole; one for a circle with center at the pole; one for a circle with center NOT at the pole; one for a circle with center located in what would be called the "second quadrant" if we were using Cartesian rather than polar labeling. Briefly describe eccentricity including examples of the equations in polar form of an ellipse, a circle, a hyperbola, and a parabola. (20 pts)

Extra Credit ----------------------------------- 5 pts ....

We all know that if $f(x) = x^2$ then $f'(x) = 2x$. If someone knows a function $g(x)$ such that $g'(x) = 2x$ but $g(x) \neq x^2$, what can be said about the difference $g(3) - g(1)$? Explain, of course.