A.P. Calculus

Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

No calculators.

Chapters 1 → 4 Exam

I The definition of continuity is actually a collection of three "tests". For each of the following, draw a sketch which illustrates the situation when only the "item" listed fails. (15 pts tot)

a) Item 1 b) Item 2 c) Item 3

II Given $\frac{1+\sin x}{1-2\sin x}=\frac{3}{4}$. Find $\cos 2x$ (15 pts)

III Given $f(x)=\begin{cases} x^2 & ; \quad x \le 1 \\ 2x-1 & ; \quad x > 1 \end{cases}$ (10 pts ea)

a) ***Prove*** $f(x)$ is continuous at $x = 1$. b) ***Prove*** $f'(1)$ does or does not exist.

IV Determine the following limits: (5 pts ea)

a) $\lim\limits_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}$ b) $\lim\limits_{x\to 0}\sqrt{1+\frac{1}{x}}-\sqrt{\frac{1}{x}}$ c) $\lim\limits_{x\to 1}\left[\frac{1}{1-x^3}-\frac{1}{3(1-x)}\right]$

d) $\lim\limits_{x\to 0}\left(\frac{\tan 3x}{\tan 2x}\right)$ e) $\lim\limits_{x\to 0}\left(1+\frac{1-\cos x}{x}\right)^{\frac{\sin x}{x}}$ f) $\lim\limits_{x\to 2}\frac{\sin(5x-10)}{3x-6}$

g) $\lim\limits_{x\to 0}\left(\frac{\pi\sec\left(x+\frac{\pi}{3}\right)-\pi\sec\frac{\pi}{3}}{x}\right)$

V Use the Intermediate Value Theorem to show that the graphs of $f(x)=x^4-5x^2$ and $g(x)=2x^3-4x+6$ intersect between $x = 3$ and $x = 4$. (15 pts)

Extra Credit ---------- 5 pts ----------

Two missiles speed directly toward each other, one at 9,000 MPH and one at 21,000 MPH. If they start at 1,317 miles apart, how far apart are they one minute before they collide?