Advanced Placement Calculus Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept*!*

No Calculators.

Chapters 1 – 4   Exam

I

Let $f(x) = \begin{cases} 1 + |x| & if & x < 0 \\ x^3 & if & 0 \le x \le 2 \\ 10 - x & if & x > 2 \end{cases}$ 　　　　　(15 pts tot)

　　a) Prove $f(x)$ is or is not continuous at $x = 0$.　　　b) Prove $f(x)$ is or is not continuous at $x = 2$

　　c) Prove $f'(0)$ exists or does not exist.　　　　　　d) Prove $f'(2)$ exists or does not exist.

II
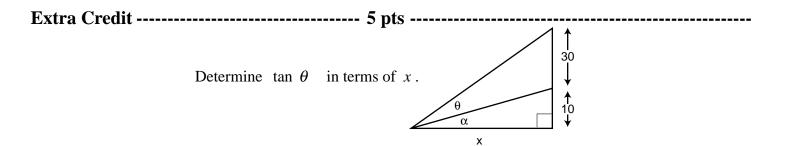
Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$ 　　　　　(25 pts tot)

　　a) Find the zeros of f.

　　b) Write the equation of the line tangent to the graph of f at $x = -1$.

　　c) The point (a, b) is on the graph of f and the line tangent to the graph at (a, b) passes through the point $(0, -8)$ which is not on the graph of f. Find the values of a and b.

III

Verify with a careful and *complete explanation* that the equation $x^5 - x^2 + 17 = 2x$ has at least one solution. 　　　　　(10 pts)

IV

Let $f(x) = \sqrt{x^2 + 9}$. Determine $g(x)$ so that $f(g(x)) = x - \sqrt{x}$. 　　　　　(15 pts)

V

Limits. Be very sure your method is justified. (Hint: All limits exist) 　　　　　(5 pts ea)

　　a) $\lim\limits_{x \to 0} \dfrac{\sin x^2}{(\sin x)^2}$　　　b) $\lim\limits_{x \to 2} \dfrac{\sin(5x - 10)}{3x - 6}$　　　c) $\lim\limits_{h \to 0} \dfrac{\cos(x + h) - \cos x}{h}$

　　d) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{2x^2 + 1}}{x + 2}$　　　e) $\lim\limits_{x \to 4} \dfrac{x^3 - 64}{\sqrt{x} - 2}$　　　f) $\lim\limits_{x \to -\infty} \dfrac{2x - 20}{\sqrt{x^2 - 100}}$　　　g) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{1 - \sin x}{\cos^2 x}$

Extra Credit ------------------------------------ **5 pts** ---

Determine $\tan \theta$ in terms of x.