A.P. Calculus

Name _____

Copy original problem.
Convince *me* that **you** understand the concept!
Each section 25 pts.
No calculators, of course.

Per _____ Date _____

Exam # 2

I Given $f(x) = \dfrac{1}{1-x}$; $x \neq 1$

A) Using the definition of the derivative, find $f'(x)$.

B) Determine the *x*-intercept of the tangent line to $f(x)$ at $x = 3$

C) Find the equation(s) of all tangent lines to $f(x)$ which are perpendicular to: $x + 4y - 2 = 0$.

II Explain 'ums

A) **Without using examples**, explain how you could use the Intermediate Value Theorem to prove that an equation had a solution.

B) Explain everything there is to know about: $\lim\limits_{x \to a} f(x) = f(a)$.

III Given $f(x)$. If possible, determine k so that $f(x)$ will be differentiable.
Prove that your value of k works.

A) $$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1} & x \neq 1 \\ k & x = 1 \end{cases}$$

B) $$f(x) = \begin{cases} x + 1 & x > 0 \\ k & x = 0 \\ x - 1 & x < 0 \end{cases}$$

IV Determine the following limits, if possible.

A) $\lim\limits_{x \to 1^+} \dfrac{\sin\sqrt{1-x}}{\sqrt{1-x}}$

B) $\lim\limits_{x \to 0} \dfrac{\sin x \cos x - 2\sin x}{x}$

C) $\lim\limits_{x \to 1} (2-x)^{\frac{1}{x-1}}$

D) $\lim\limits_{x \to \pi} \dfrac{|\sin x|}{x - \pi}$

E) $\lim\limits_{x \to 3^-} \dfrac{|x+3|}{\sqrt{x}}$

Extra Credit ---------- 5 pts ----------

Let $f(x) = \sqrt{x^2 + 9}$. Determine two different functions $g(x)$ so that $f(g(x)) = x - \sqrt{x}$.