Advanced Placement Calculus

Name _____

Copy original problem.

Per _____   Date _____

Convince *me* that **you** understand the concept*!*

Quarter # 1   Exam # 1

I   Given $f(x) = \sqrt{\dfrac{x^2 - x - 2}{x^2 + x - 6}}$. Determine the domain of $f(x)$. Justify.   (10 pts)

II   Given $f(x) = 3x^4 + 2x^3$. Determine the equation of the tangent line to $f(x)$ at $(-1, 1)$.   (10 pts)

III   Solve $\forall$ $x \, \varepsilon \, \Re$. Graph solution set on a well labeled number line. Be especially sure that the reasons for your steps are very clear.   (10 pts ea)

   a) $\left| \dfrac{x - 2}{x + 2} \right| > 1$ b) $\dfrac{5}{1 - x} < 3$

IV   Using the definition of the derivative, determine $f'(1)$ where $f(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ \sqrt{x} - 1 & \text{if } x \geq 1 \end{cases}$ (10 pts)

V   Given $f(x) = |2x + 3| - |4 - x|$.   (15 pts tot)

   a)   Rewrite $f(x)$ as a piece-wise function.

   b)   Sketch the graph of $f(x)$.

   c)   Determine $f'(x)$

VI   Find the following limits: (5 pts ea)

   a) $\displaystyle\lim_{x \to 0^-} \dfrac{2x^2 + 3x}{|x|}$ b) $\displaystyle\lim_{x \to -3} \dfrac{x^3 + 27}{x + 3}$ c) $\displaystyle\lim_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$

   d) $\displaystyle\lim_{x \to 0} \dfrac{x^2 + \frac{1}{x}}{x^2 - \frac{1}{x}}$ e) $\displaystyle\lim_{x \to 0} (1 - 3x)^{\frac{2}{x}}$ f) $\displaystyle\lim_{x \to \infty} \sqrt{x^2 + x} - \sqrt{x^2 + 4}$

   g) $\displaystyle\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} 4 - x & \text{if } x < 1 \\ \sqrt{6} & \text{if } x = 1 \\ 2 + x^2 & \text{if } x > 1 \end{cases}$

Extra Credit ----------------------------------- **5 pts** ---

   A ball was floating in a lake when the lake froze. The ball was removed (without needing to break the ice), leaving a hole 24 inches across at the top and 8 inches deep at the deepest point of the indentation. What was the radius of the ball?