

Advanced Placement Calculus

Name _____

Copy original problem.

Per _____

Date _____

Convince *me* that **you** understand the concept!*No calculators.*

First Exam

I Given $f(x) = \frac{2x-1}{x+2}$, $g(x) = \frac{2x-1}{x-3}$ (5 pts ea)

a) $B(x) = \frac{1}{\frac{1}{f(x)} + \frac{1}{g(x)}}$. Find D_B

b) $C(x) = \sqrt{-\frac{f(x)}{g(x)}}$. Find D_C

II Solve for all $x \in \mathfrak{R}$. Graph solution on a number line. (10 pts ea)

a) $\frac{|x+2|}{|x-3|} \geq 1$

b) $\frac{1}{2x-3} \leq 4$

III The “definition of the derivative” is generally written in one of two forms. Both forms require the use of limits. (15 pts total)

- a) State **BOTH** of these “definitions.”
- b) Using one of your responses in part A, PROVE that $f'(-2)$ does not exist when $f(x) = |x+2|$
- c) Using the other of your responses in part A, find the derivative of $f(x) = 2x^2 + 3$

IV Given: $f(x) = |2x-3| - |x-2|$ (10 pts total)

- a) Write $f(x)$ as a piece-meal function.
- b) Determine $f'(x)$.

V Evaluate the following limits, if they exist. Be sure your method is clear to me. If a limit does not exist, explain the reason it does not. (5 pts ea)

a) $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 4}}{x+2}$

b) $\lim_{x \rightarrow \infty} \frac{-2x+4}{9x^2+6}$

c) $\lim_{x \rightarrow \infty} \sqrt{x^2+x} - \sqrt{x^2+4}$

d) $\lim_{x \rightarrow -3} \frac{x^3+27}{9-x^2}$

e) $\lim_{x \rightarrow 9} \frac{6-2\sqrt{x}}{x-9}$

f) $\lim_{x \rightarrow 0^+} x \sqrt{\frac{4}{x^2} - 1}$

g) $\lim_{x \rightarrow 0} \frac{|x|-x}{x}$

h) $\lim_{x \rightarrow -3} |-2|$

i) $\lim_{x \rightarrow 8^-} \frac{\sqrt{x}}{4}$

Extra Credit ----- 5 pts -----

Given $P > \sqrt{a}$ and $Q = \frac{P + \frac{a}{P}}{2}$. Prove: $P > Q$