Mr. Mumaugh

$f \quad f' \quad f''$

I

Given: $f(x) = x^4 - 2x^3 + x^2$

$f''(x) = 0 \rightarrow x \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$

$f'\left(\frac{1}{4}\right) \approx \frac{1}{2}; \quad f'\left(\frac{3}{4}\right) \approx -\frac{1}{2}$

$f''(x)$ has minimum at $\left(\frac{1}{2}, -1\right)$

1. Express $f(x)$ in factored form; determine ***all*** zeros of $f(x)$. Plot those points on the top axis.

2. Find $f'(x)$ and write in factored form.

3. Find all critical values of x. $f'(x) \overset{\text{set}}{=} 0$ and solve. Plot the zeros on the middle axis.

4. Write the **coordinates** of all critical values. Plot on the top axis.

5. Find $f''(x)$. Normally you would find these zeros now. However, the zeros were given above. We shall use them. Plot zeros on bottom axis.

6. Put values found in #5 into $f(x)$. Write coordinates. Plot on top axis.

7. Put values found in #5 into $f'(x)$. Write coordinates. Plot on middle axis.

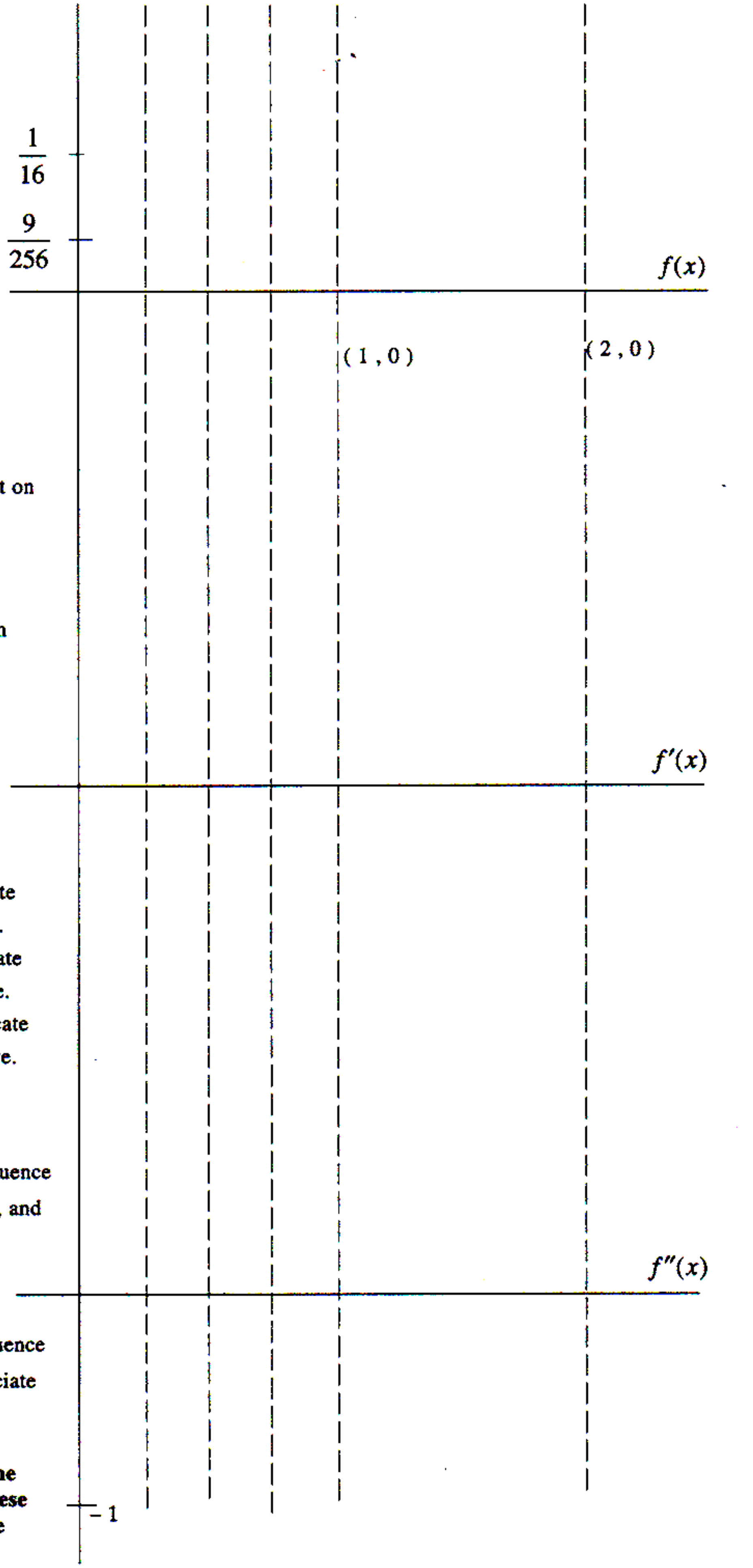
8. On a number line, locate zeros of $f(x)$. Indicate where $f(x)$ is positive and where it is negative. On a number line, locate zeros of $f'(x)$. Indicate where $f'(x)$ is positive and where it is negative. On a number line, locate zeros of $f''(x)$. Indicate where $f''(x)$ is positive and where it is negative.

9. Sketch all three functions on their axes.

10. Explain how the values (+ or –) of $f''(x)$ influence $f(x)$ and $f'(x)$. i.e. What behavior in $f(x)$, and $f'(x)$ do you associate with $f''(x) > 0$? ; $f''(x) > 0$?

11. Explain how the values (+ or –) of $f'(x)$ influence $f(x)$ i.e. What behavior in $f(x)$ do you associate with $f'(x) > 0$? ; $f'(x) > 0$?

12. **The results of the above 11 problems are of the utmost importance !! Write a summary of these results AND your conclusions. Use complete sentences and be VERY complete.**



II

Carefully draw F ' and F " on the graphs.

Show the corresponding points of A, B. C, D, etc

Justify the locations of A', B' C' etc using complete sentences.

Determine the DEGREE of F(x).

(hint : $f'(x)$ and $f''(x)$ are continuous.)

III

Sketch a smooth curve of F (x) illustrating the following :

(Each problem uses a unique axis system and is independent of any other problem.)

a) $f(1) = 3,\ \ f' < 0\ \forall\ x < 1;\ f' > 0\ \forall\ x > 1$

b) $f(1) = 3,\ \ f'' < 0\ \forall\ x < 1;\ f'' > 0\ \forall\ x > 1$

c) $f' > 0\ \forall\ x < 2;\ f' < 0\ \forall\ x > 2;\ f(2) = 3$

d) $x > 0\quad f(1) = 0;\ f'(x) = \frac{1}{x}$

(hint : What is $f''(x)$ and what does it tell you?)
Use the def of the derivative to find out . . .

IV

Given: $f(x) = \frac{x^2 + 1}{x}$

$D_f \quad -2 \le x \le 2;\ \ x \ne 0$

A) Find $f'(x)$, using the second definition of the derivative.

B) Find $f''(x)$, using the second definition of the derivative.

C) Make a number line for $f(x)$.

D) Make a number line for $f'(x)$.

E) Make a number line for $f''(x)$.

F) What are the coordinates (of $f(x)$) of the critical values.

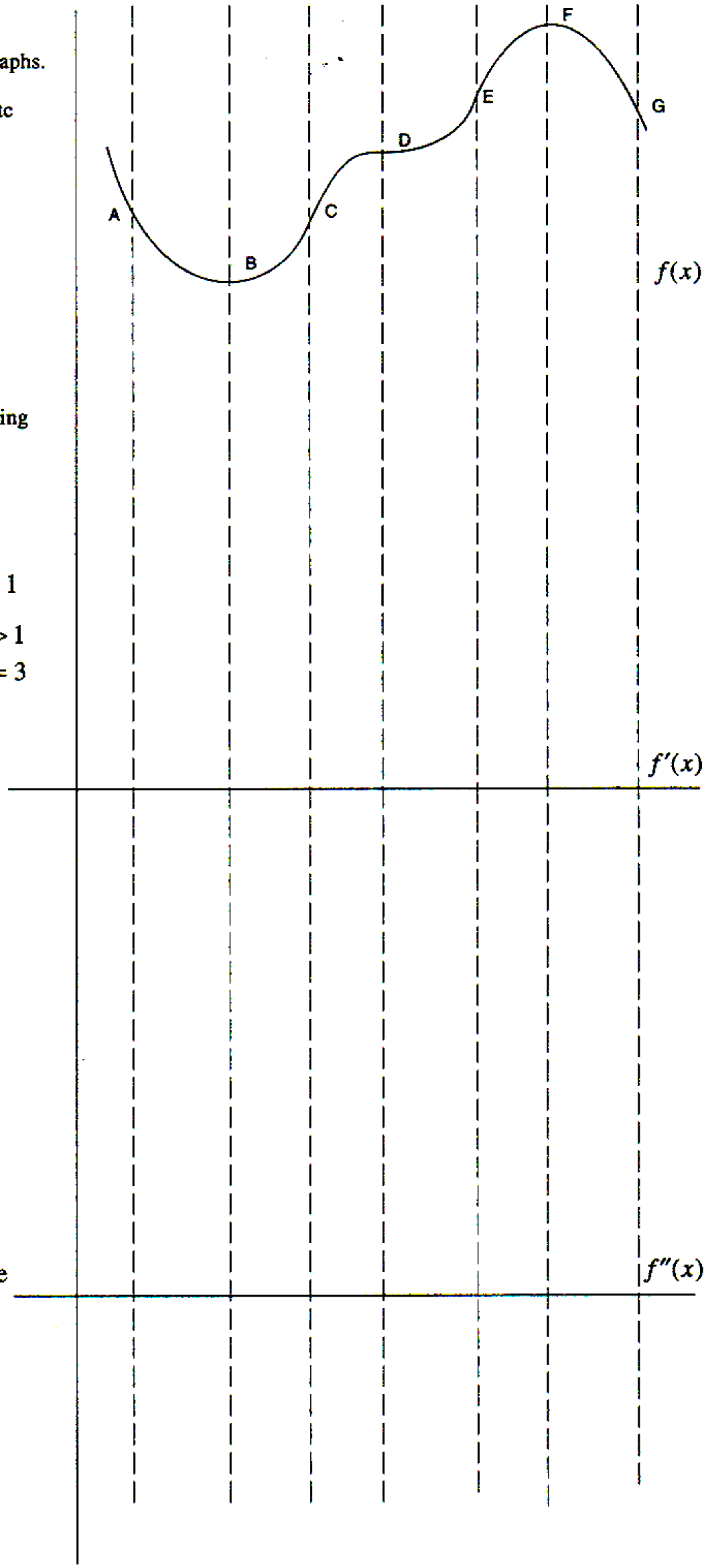
G) What are the coordinates of (of $f(x)$) $(-2, f(-2))$ and $(2, f(2))$?

H) List the coordinates and specifically identify all extrema.

I) List intervals where $f(x)$ is increasing.

J) List intervals where $f(x)$ is concave up.

K) Sketch $f(x)$



El Quizzo

No Calculators

Name ____________________

Per ______ Date __________

I Given $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$ for $-1 \le x \le 2$.

Determine and identify all significant points (ie. extrema, PI, intercepts, etc)

II Find the limit:

a) $\lim_{x \to 0} \left(\frac{3+2x}{x+5x^2} - \frac{3}{x} \right)$

b) $\lim_{x \to 0} \frac{\sqrt[3]{x+1} - 1}{x}$

Turn over for part III

III State and identify both "definitions of the derivative."

Explain how they are different.
Explain how they are the same.
Explain when each is used. Be complete.
Demonstrate each definiton of the derivative

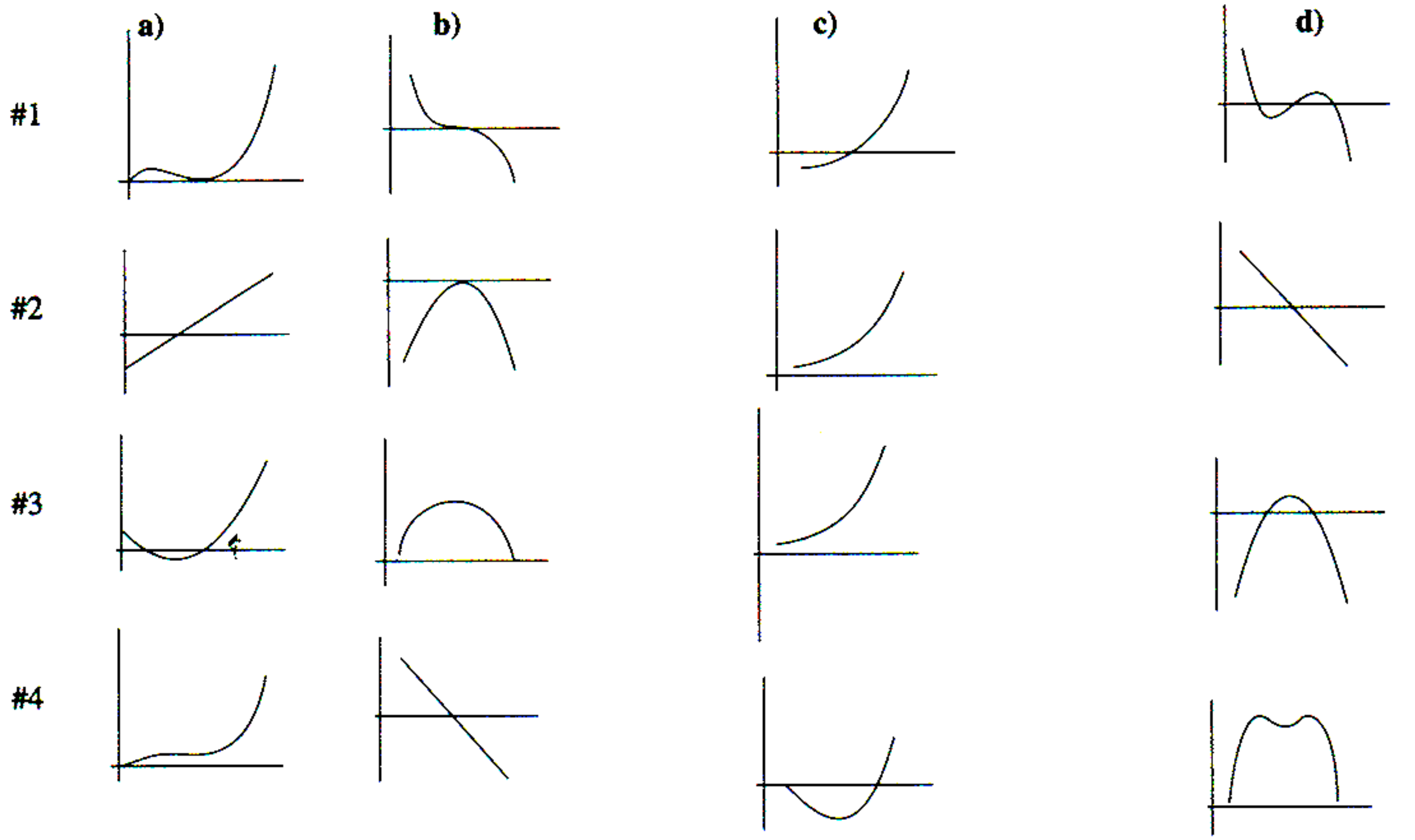
using the function $f(x)=\dfrac{x^2+1}{x}$ and the point $\left(2,\dfrac{5}{2}\right)$

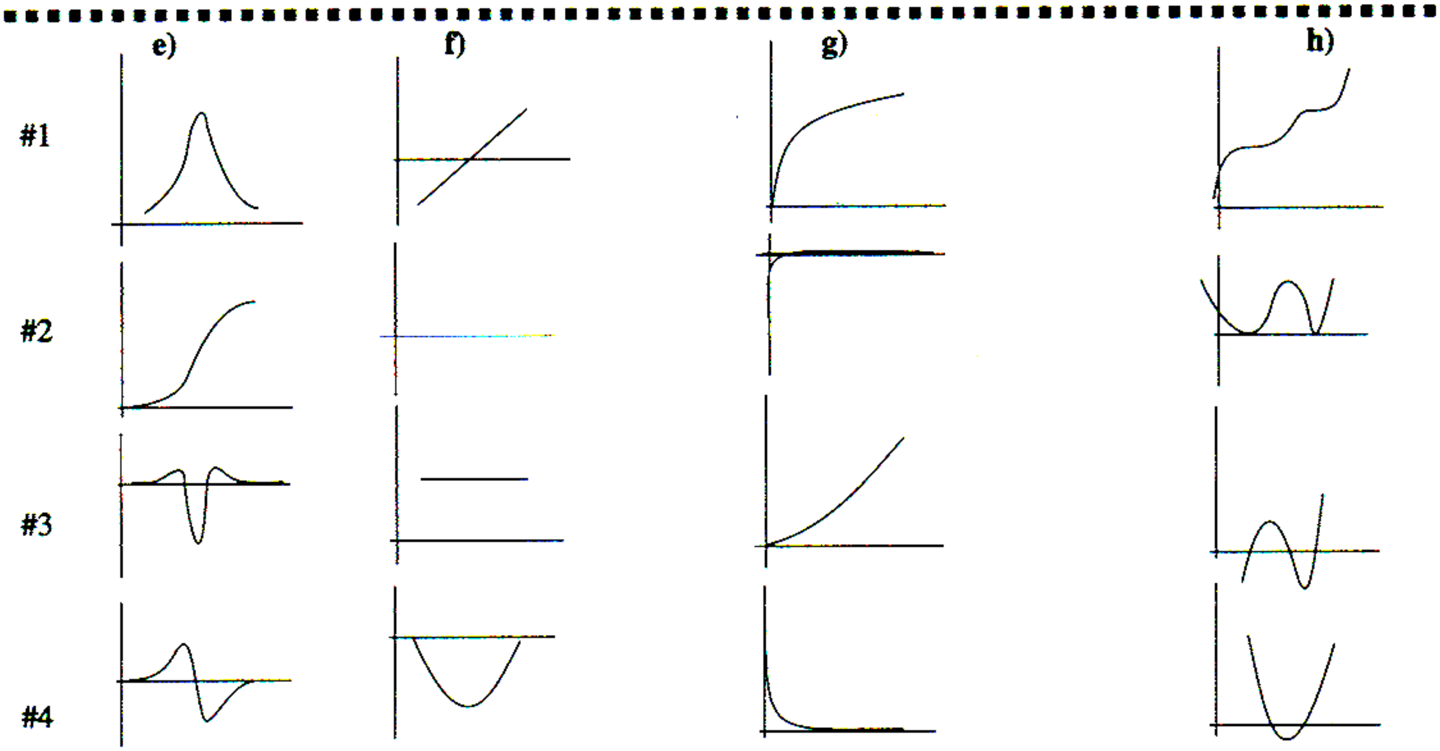
Analysis

J Mumaugh

We want *contenders!* Can you find the *pretender*?

Each of the following columns is headed by the graph of a function $f(x)$. Underneath this graph are three others that are, in random order, the graphs of $f'(x), f''(x)$, and another function. Your job is to determine the pretender and to label the others properly.

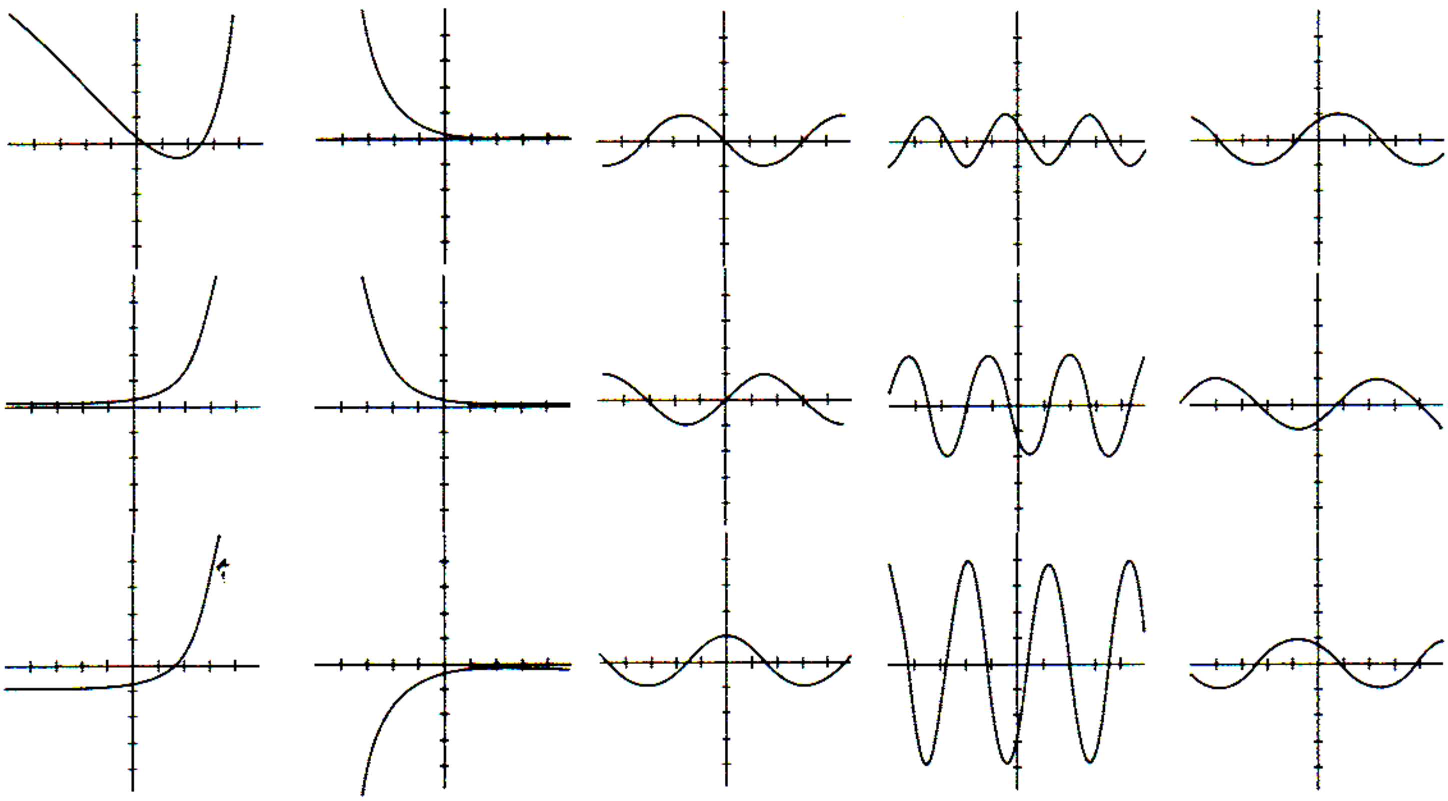




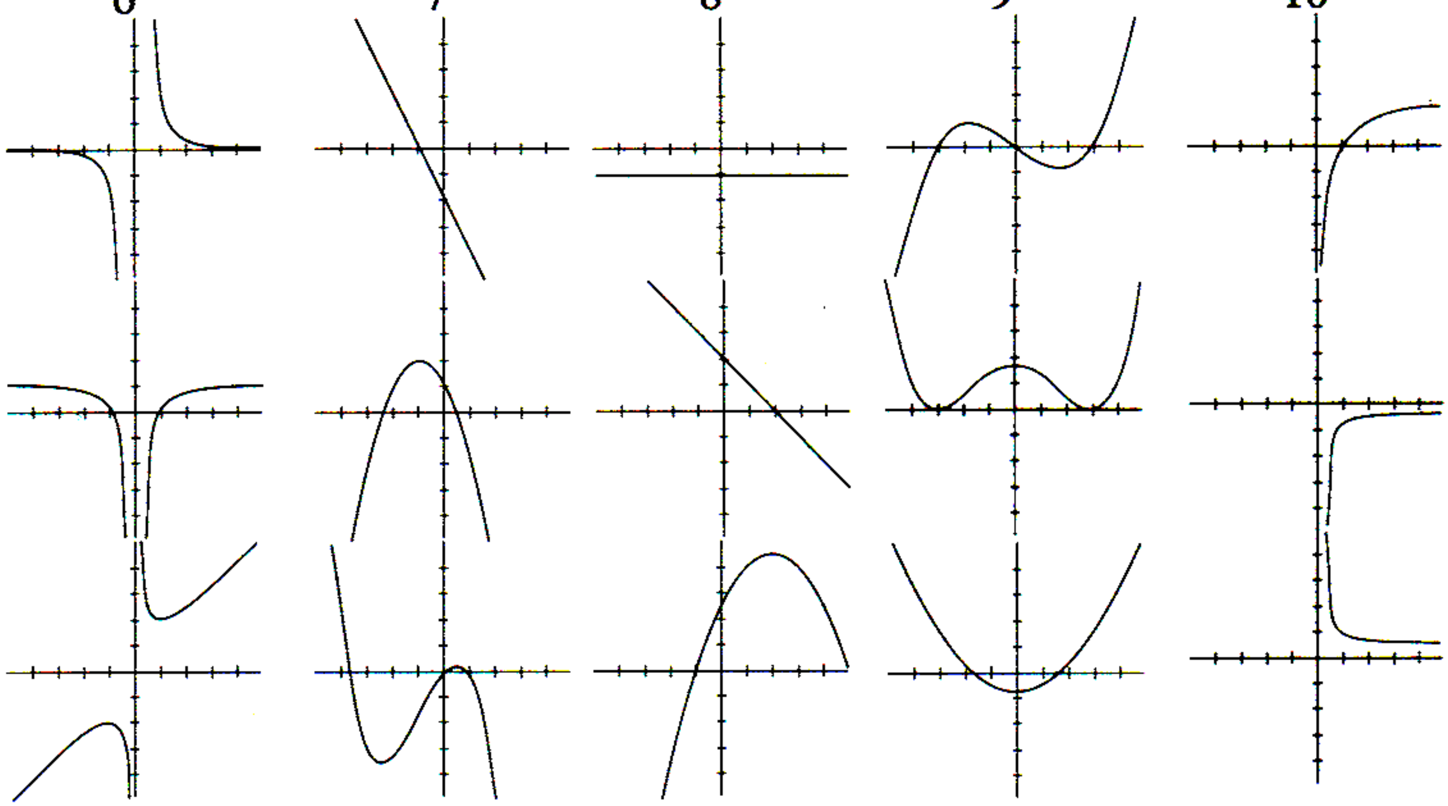
Sherlock Heston Knows his f from his f' and his f''. Do you?

Under each number are 3 axes. The curves, f , f', and f'' are in random order. Your job is to decide which curve is which. Consider problem #1. The first graph is f and it is concave up everywhere so the 2nd curve would be f'' since it is everywhere positive and the third curve changes from negative to positive when f changes from decreasing to increasing so it would be f'.

1 2 3 4 5



6 7 8 9 10



Honors Analysis   Chapter 3   Heavy Duty Quiz   Name ____________________

Open Notes   NOT open friends   **25 minutes**   Period ______   Date ________

Do all work and put answers on this paper.

I	**II**	**III**	**IV**

I   What is the effect on:

a) $f(x)$ and $f'(x)$ when $f''(x) > 0$

---

b) $f(x)$ when $f'(x) = 0$

---

c) $f(x)$ when $f'(x) < 0$

---

d) $f(x)$ when $f'(x) > 0$

---

e) $f(x)$ and $f'(x)$ when $f''(x) < 0$

---

f) $f(x)$ and $f'(x)$ when $f''(x) = 0$

II   Given $f'(x)$, and $f''(x)$ number lines. Sketch the graph of a function which has the properties described:

a)

	A	B	C	D	E
$f'(x)$	O		O	+	⊗
$f''(x)$	−	O		O	O

A   B   C   D   E

b)

	A	B	C	D	E
$f'(x)$	− O		⊗		O
$f''(x)$	+	O	O	O	

A   B   C   D   E

c)

	A	B	C	D	E
$f'(x)$			+		
$f''(x)$	O	O	O	−	O

A   B   C   D   E

Sections III and IV on back.

III Find the limit: $\lim_{x \to 1}\left(\frac{1}{1-x^3} - \frac{1}{3(1-x)}\right)$

IV Given: $f(x) = 3x^4 - 8x^3 + 6x^2 + 1 \quad \forall \; x \; \varepsilon \; [-1, \; 2]$.

Determine (identify as local or global) the *coordinates* of all extrema and PI. Justify, of course.

Determine if the following statements are true or false. Explain reasoning. A counter-example is acceptable (you still need to explain your method.)

1) $f(x)=\begin{cases}\dfrac{x^2+2x-3}{x+3} & \text{if } x\neq -3\\ -4 & \text{if } x=-3\end{cases}$ $f(x)$ is continuous at $x=-3$.

2) Let $f(x)$ be a function, D_f $x \varepsilon R$.

a) If $f(x)$ is differentiable (has a derivative) at $x=2$, then $f(x)$ must be continuous at $x=2$.

b) If $f(x)$ is continuous at $x=2$, then $f(x)$ must be differentiable at $x=2$.

c) If $f(x)$ is continuous at $x=2$, then $\lim_{x\to 2}(f(x)-f(2))=0$.

d) If $f(x)$ is continuous, then $|f(x)|$ is continuous. (T)

e) If $|f(x)|$ is continuous, then $f(x)$ is continuous. (F)

f) If $f(x)$ is continuous, then $f(|x|)$ is continuous. (T)

3) If $f'(x)=g'(x)$, then $f(x)=g(x)$.

4) Let $\lim_{x\to a} f(x) = f\left(\lim_{x\to a} x\right)$.

a) $f(x)$ is continuous.

b) $f(x)$ is differentiable.

Provide a detailed reason for each problem:

5) Define continuity. Give the "Dick & Jane" version and the mathematical version. Prove $f(x)=x^3-8$ is continuous at $x=2$.

6) Write the two versions of the definition of the derivative.

a) Explain how they are different.

b) Explain how they are the same.

c) Show a properly labeled graphical representation for each (as was done in lecture).

d) Tell which version is best suited for which purposes.

e) Using each of the versions, find $f'(2)$ when $f(x)=x^3-8$.

7) Given $f(x)=|x-3|$.

a) Prove $f(x)$ is continuous at $x=3$.

b) Prove $f'(3)$ DNE.

8) Draw the graph of a function which illustrates the situation concerning the definition of continuity:

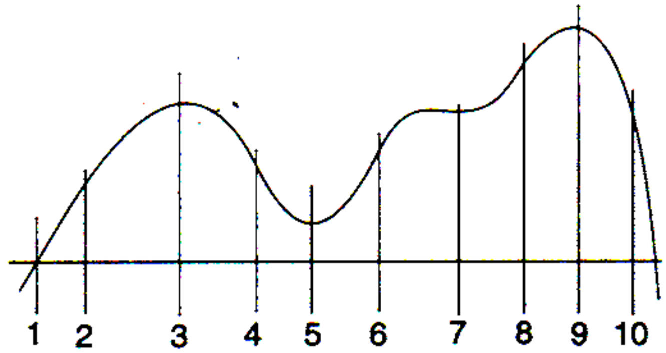
a) Step 1 met. Step 2 met. Step 3 not met.

b) Step 1 not met. Step 2 met.

c) Step 1 not met. Step 2 not met.

d) Step 1 met. Step 2 not met.

9. Use the figure to the right to determine "True" or "False".



a) $f(x)$ changes sign at $x = 1$.

b) $f''(2) = 0$

c) $f'(3) = 0$

d) $f''(4) = 0$

e) $f''(5) = 0$

f) $f''(5) < f''(3)$

g) $f'(7) < 0$

h) $f''(8) = 0$ i) $f(2) < f(7)$ j) $f'(9) = 0$ k) $f'(10) = 10$ l) $f''(10) < 0$

m) $f'(2) < f'(4)$ n) $f'(7) < f''(7)$ o) $f''(2) > f''(4)$

10. Given: $f(x) = \begin{cases} 2x+1 & \text{if } x \,\varepsilon\, [-2,0) \\ -x+1 & \text{if } x \,\varepsilon\, (0,2) \\ -1 & \text{if } x = 2 \\ 2x-5 & \text{if } x \,\varepsilon\, (2,3] \end{cases}$

> Note: $x \;\varepsilon\; [-2,\ 0)$
> means the same thing as
> $-2 \le x < 0$
>
> The bracket means "including.."
> and the parenthesis means "not including"

a) Prove f is or is not continuous at $x = 0$.

b) Prove f is or is not continuous at $x = 2$.

c) Prove $f'(0)$ does or does not exist.

d) Prove $f'(2)$ does or does not exist.

e) Draw the graph of f.

11. Given: $f(x) = 3x^4 - 4x^3$; $D_f\, x \;\varepsilon\; [-1,2]$

a) Determine all zeros of f.

b) Find $f'(x)$ and $f''(x)$

c) Draw number lines for $f'(x)$ and $f''(x)$.

d) Compute the coordinates of all significant points. (Justify, of course)

e) Graph f.

12. Given: $f(x) = \begin{cases} \dfrac{x^3-1}{x-1} & \text{if } x \ne 1 \\ k & \text{if } x = 1 \end{cases}$

a) Determine the value of k so f is continuous.

b) Find $f'(1)$

13. Given: $f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ k & \text{if } x = 1 \\ 2x & \text{if } x > 1 \end{cases}$

a) Determine the value of k so f is continuous.

b) Find $f'(1)$

Mr. Summary

Mr. Mumaugh

Use a calculator where *necessary* (as contrasted by use due to *laziness*).

I The limit concept

A) Explain, compare and contrast the following: $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a} f(x)$, $\lim_{x\uparrow a} f(x)$, $\lim_{x\downarrow a} f(x)$

B) Explain what "If there is no reason not to, we just stuff it in." means and why and where do we use it.

C) Explain how to "simplify" limit problems. Discuss various methods *but do not use* any specific examples.

II Explain the effect on ....

A) $f(x)$ when $f'(x)>0$

B) $f(x)$ when $f'(x)<0$

C) $f(x)$ when $f'(x)=0$

D) $f(x)$ when $f''(x)=0$

E) $f(x)$ when $f''(x)>0$

F) $f(x)$ when $f''(x)<0$

G) $f'(x)$ when $f''(x)<0$

H) $f'(x)$ when $f''(x)>0$

I) $f'(x)$ when $f''(x)=0$

III Given: $f(x)=\dfrac{x^2+16}{x-3}$ and $f'(x)=\dfrac{x^2-6x-16}{x^2-6x+9}$; also $g(x)=x+3$; $h(x)=f(x)-g(x)$

A) Using long division divide the denominator of $f(x)$ into the numerator of $f(x)$.

B) Explain why there is a vertical asymptote for $f(x)$ at $x=3$. (Describe the situation as $x\to 3^-$ for example.) Be complete! Remember, this *is* an honors class so do honors level work!

C) Compute (maybe you could make your *programmable* calculator do this for you?)

1) Find $f(8)$, $f(80)$, $f(800)$, $f(8000)$ and $f(-2)$, $f(-20)$, $f(-200)$, $f(-2000)$.

2) Find $h(8)$, $h(80)$, $h(800)$, $h(8000)$ and $h(-2)$, $h(-20)$, $h(-200)$, $h(-2000)$

D) State a conclusion about $f(x)$ as x gets very large (and when x gets very negative). What function does $f(x)$ almost duplicate?

E) Make up number lines for $f(x)$ and $f'(x)$.

1) Specifically state the interval(s) where $f(x)$ is above the x-axis.

2) Specifically state the interval(s) where $f(x)$ is increasing.

3) Are your intervals for (1) and (2) the same? Why or Why not?

F) Determine, identify and justify the coordinates of any extrema of $f(x)$.

G) On a single axis...

1) Draw the line $x=3$ (dotted)

2) Draw the line $y=x+3$.

3) Plot the points from part (F).

4) Draw $f(x)$

5) Does your picture agree with your answers to part E (1) and (2)?

IV Given $f(x)=x^4-4x^3+3$ and $[-1,4]$

A) Find the coordinates of all extrema. (Justify, of course.)

B) Sketch $f(x)$, $f'(x)$, and $f''(x)$ on a single y-axis with 3 different x-axes.

V Given $f(x)$ is continuous at $x = a$. Only one of the following may be false. Determine which one may be false and draw a picture of a function which satisfies all of the others except that one you have picked.

i) $\lim_{x \to a} f(x)$ exists ii) $\lim_{x \to a} f(x) = f(a)$ iii) $f(a)$ and $f'(a)$ exist iv) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

VI The derivative of $f(x) = \frac{1}{3}x^4 - \frac{1}{5}x^5$ attains its maximum value(s) at what value(s) of x ?

VII The line $3x - 4y = 0$ is tangent in the first quadrant to $y = x^3 + k$. Find k.

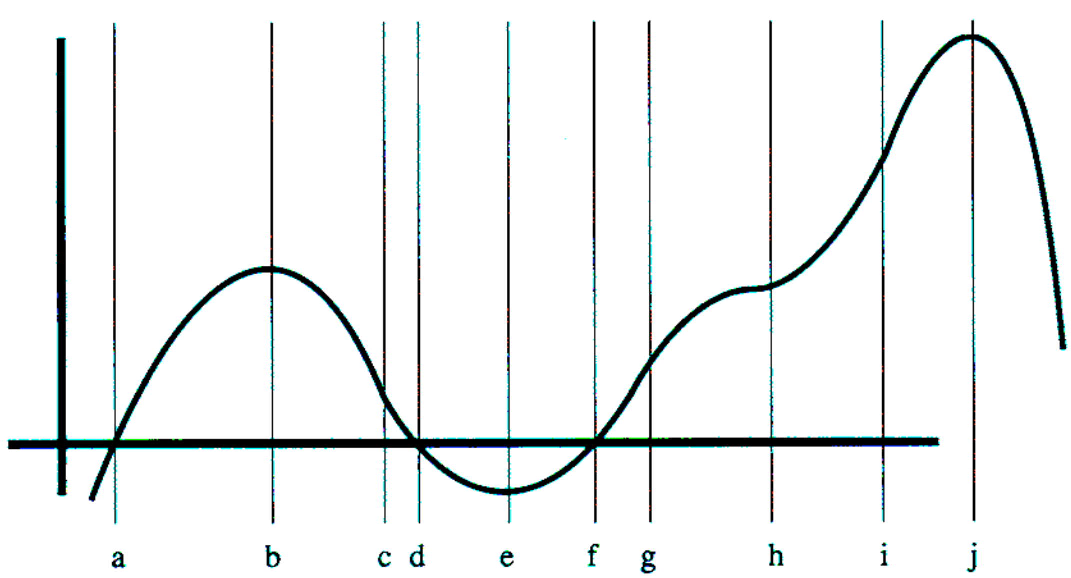
VIII Given $f(x) = 3x^5 - 20x^3$. Sketch. Determine extrema. (Justify, of course)

IX Given $f(x) = x^3 - x^2 - 4x + 4$. Find the equation of the tangent line to $f(x)$ at $x = -1$.

X True or False (and explain): $f(x) = 5x^4 - x^5$ has two inflection points.

XI Using words only, explain the three step process required to prove that a function is continuous at a particular point. Give picture examples of functions which illustrate when one or more of the "parts of the definition of continuity" has been violated.

XII The following picture is the graph of $f'(x)$. Note it is *not* the graph of $f(x)$.
Draw the pictures of $f(x)$ and $f''(x)$.
Specifically state coordinates of significant items. Justify, of course.



Mr. Mumaugh
Analysis Ch 3

Number Lines– Я –Us

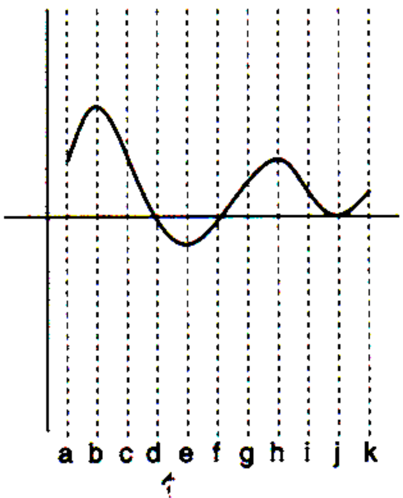
I On the number line sheet provided, **mark** the f f' f'' number lines.

On separate paper, describe all max, min and PI for $f(x)$ and include proper justification.

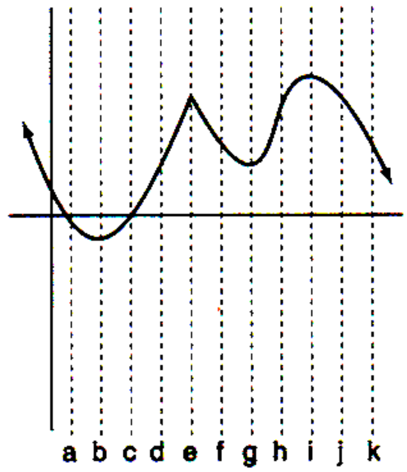
(for example: $(e, f(e))$ max f' test or

$(a, f(a))$ min End Pt. increasing from it.)

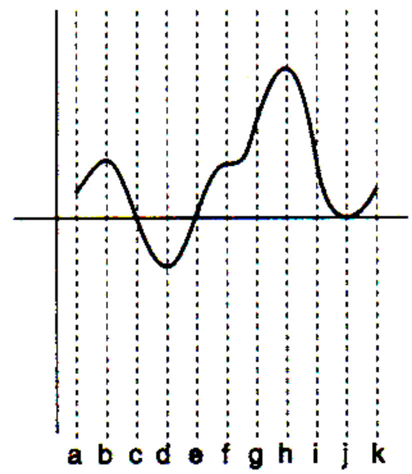
a)



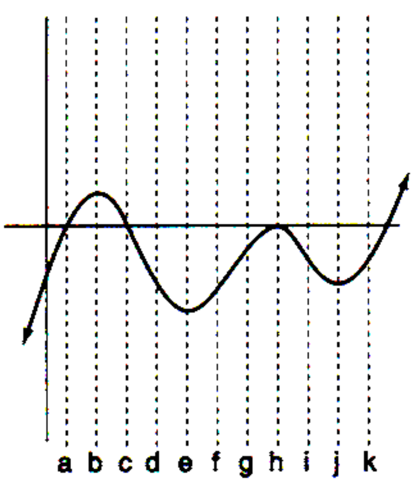
b)



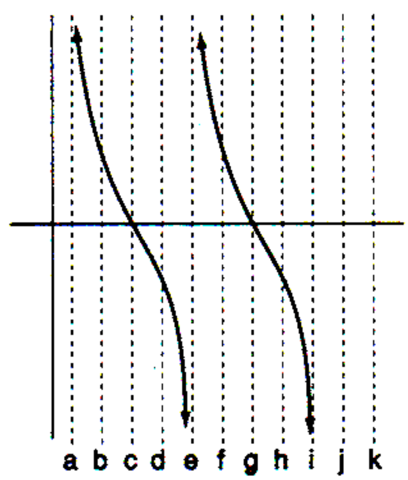
c)



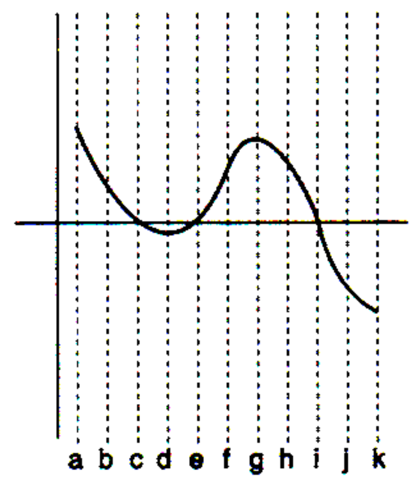
d)



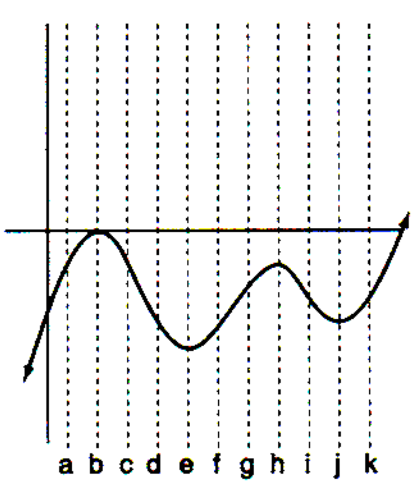
e)



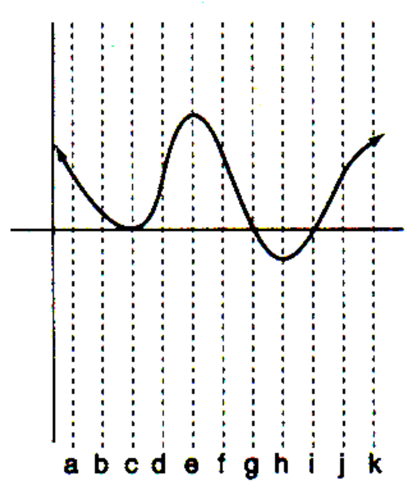
f)



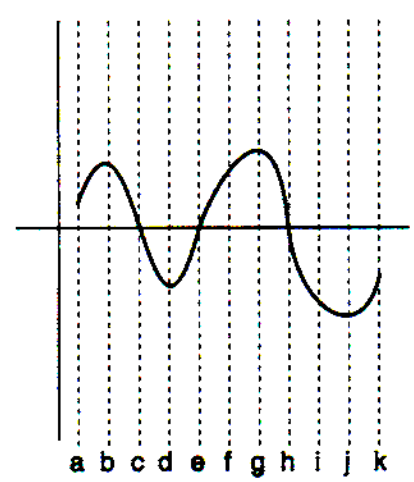
g)



h)



i)



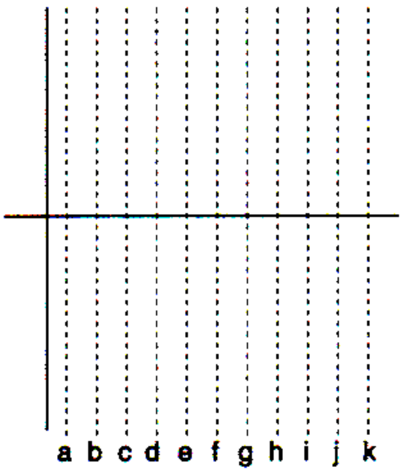
II Use the marked number lines for clues which will allow you to sketch ***only*** the graph of the function, $f(x)$.

On separate paper, describe all max, min and PI and include proper justification.

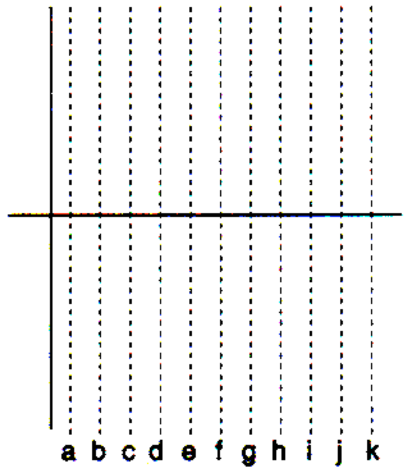
(for example: $\big(e, f(e)\big)$ max f' test or

$\big(a, f(a)\big)$ min End Pt. increasing from it.)

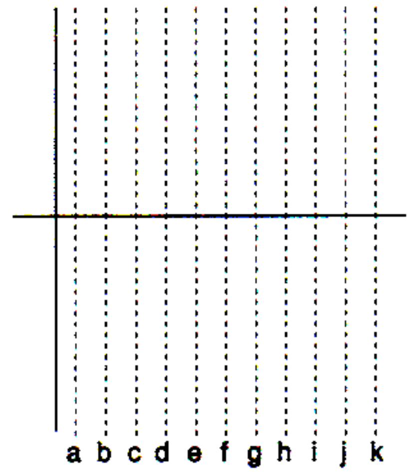
a)



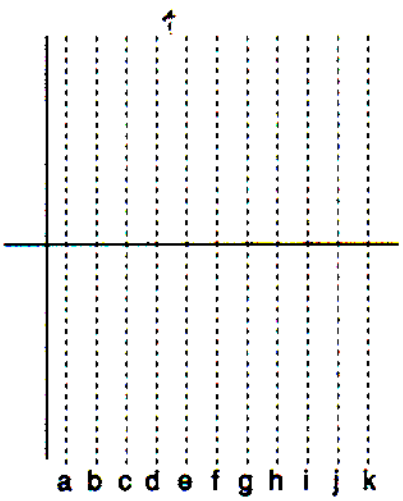
b)



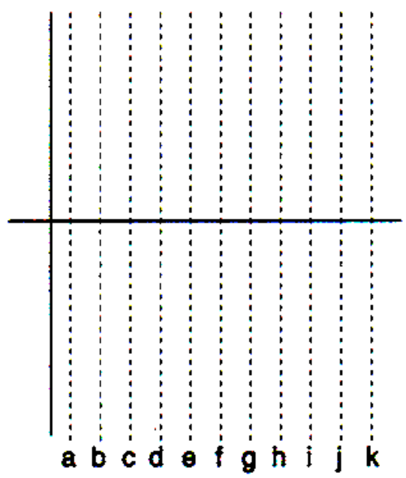
c)



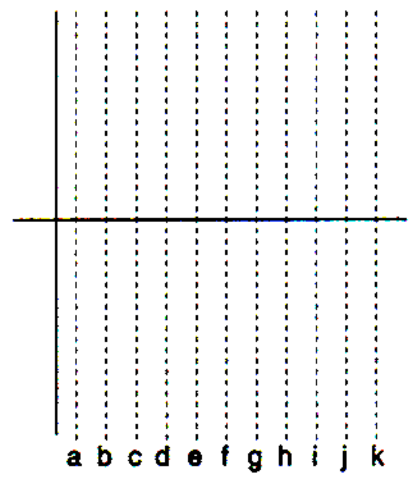
d)



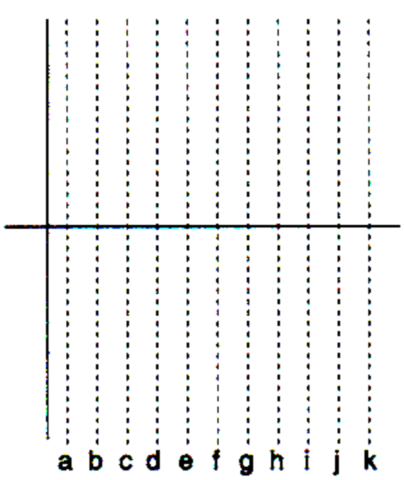
e)



f)



g)



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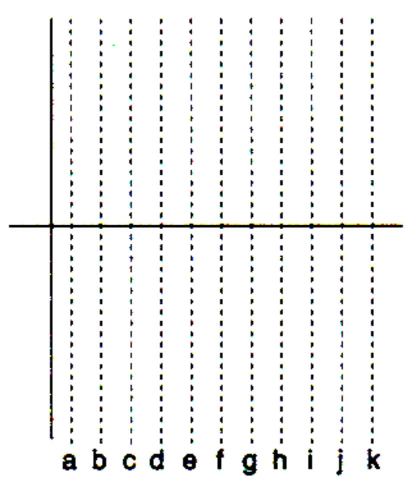
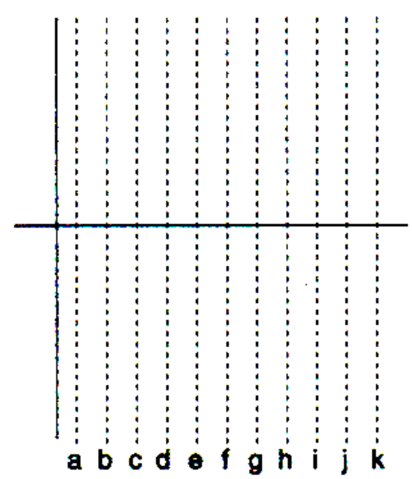
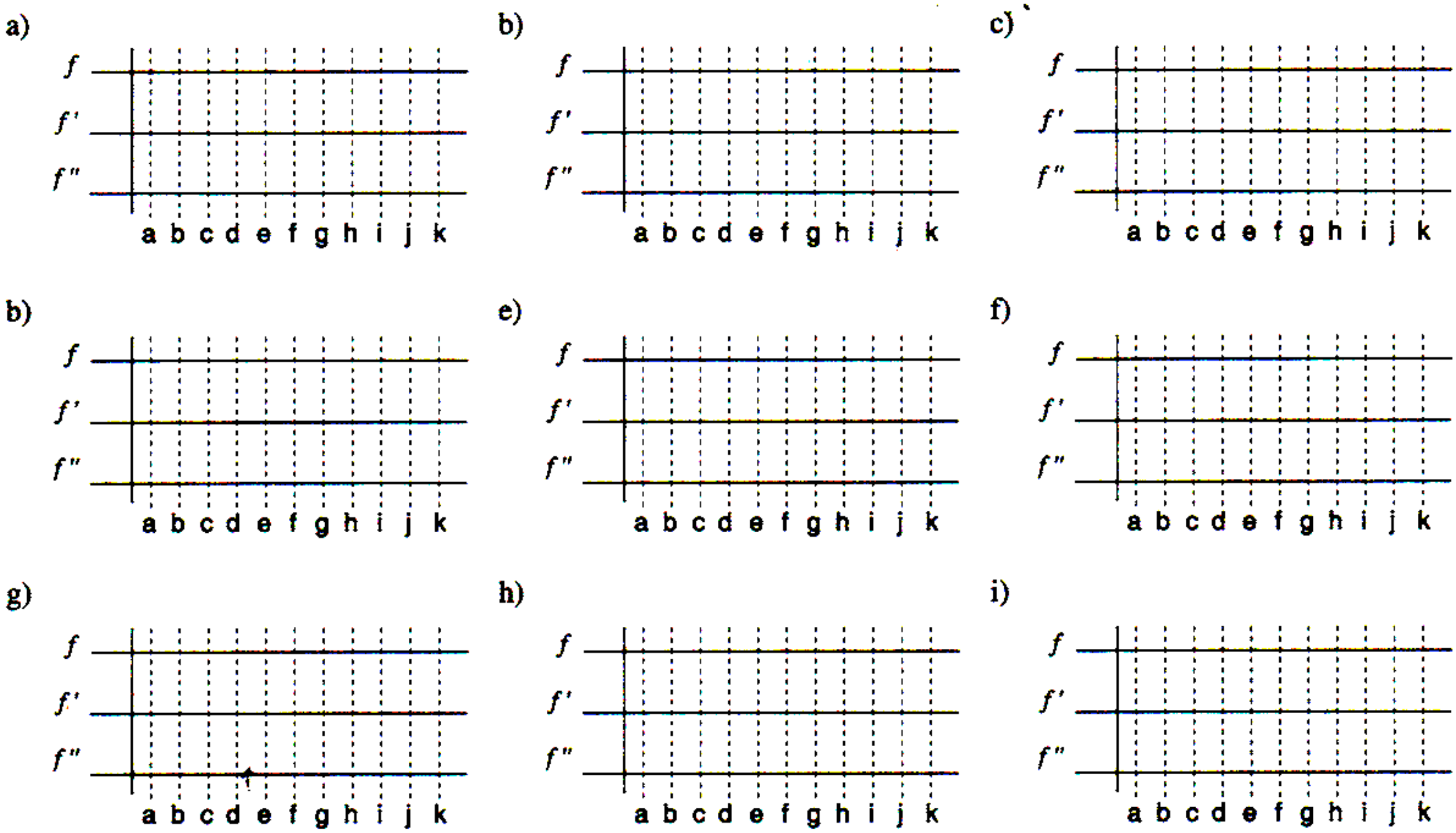


i)

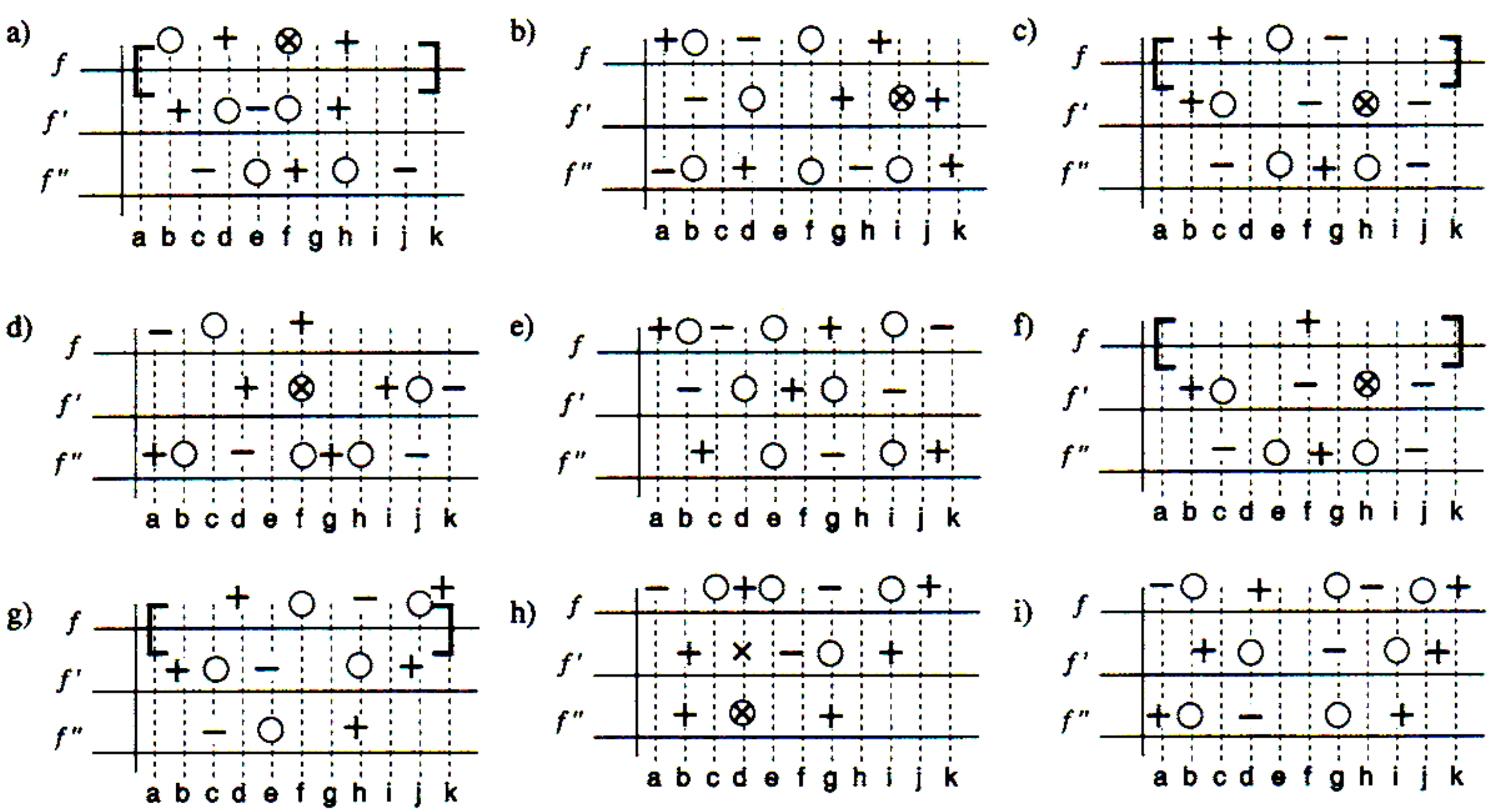


I continued ...

○ = a zero ⊗= Non Participating Point ×= DNE at that point



II continued...



unLIMITed Enjoyment

Honors Analysis

Mr. Mumaugh

I Find the following limits:

a) $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$ b) $\lim_{h \to 0} \frac{h^2}{6-\sqrt{36-h^2}}$ c) $\lim_{x \to 1} \frac{\frac{1}{x}-1}{x-1}$ d) $\lim_{x \to 1} \frac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{2}}}{x+2}$

e) $\lim_{h \to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$ f) $\lim_{x \to 1} \frac{\sqrt{x+3}-2}{x-1}$ g) $\lim_{h \to 0} \frac{(a+h)^2-a^2}{h}$ h) $\lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}$

i) $\lim_{x \to -2} (x+3)^{99}$ j) Find the limit **as h goes to zero** for problems 38, 39, 40 on page 11 in the text.

k) Clear parentheses: $(a^2 - ab + b^2)(a + b)$ l) Clear parentheses: $(p-q)(p^2 + pq + q^2)$

m) Show long division: $2x^2 + 5x - 3\overline{)6x^3 + 19x^2 + x - 6}$ n) $\lim_{x \to -3} \frac{6x^3+19x^2+x-6}{2x^2+5x-3}$

o) $\lim_{x \to -\frac{2}{3}} \frac{6x^3+19x^2+x-6}{2x^2+5x-3}$ p) $\lim_{x \to \frac{1}{2}} \frac{2x^2+5x-3}{6x^3+19x^2+x-6}$

II Find the following limits.

a) $\lim_{x \to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x}$ b) $\lim_{x \to 0} \frac{1-\sqrt{2x^2+x+1}}{x}$ c) $\lim_{x \to 2} \frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ d) $\lim_{x \to 1} \frac{x^2-2x-3}{x^2-1}$

e) $\lim_{x \to 1} \frac{x^2+2x-3}{x^2-1}$ f) $\lim_{h \to 0} \sqrt{1+\frac{1}{h}}-\sqrt{\frac{1}{h}}$ g) $\lim_{x \to -2} \frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x+2}$ h) $\lim_{x \to 2} \frac{x^4-16}{x-2}$

i) $\lim_{h \to 0} \frac{(x+h)^2-x^2}{h}$ j) $\lim_{x \to 0} \frac{\sqrt[3]{x+1}-1}{x}$ k) $\lim_{x \to -2} \frac{\frac{1}{\sqrt{x^2}}-\frac{1}{2}}{x-2}$ l) $\lim_{h \to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

m) $\lim_{x \to 1} \left(\frac{1}{1-x^3} + \frac{1}{3(x-1)}\right)$ n) $\lim_{h \to 0} \left(\frac{(x+h)^3 - x^3}{h}\right)$

III Find the limits:

a) $\lim_{x \to 2^-} \sqrt{2-x}$ b) $\lim_{x \to 0^+} \sqrt{x}$ c) $\lim_{x \to 0^-} \sqrt{x}$ d) $\lim_{x \downarrow 1} \frac{x-1}{|x-1|}$

e) $\lim_{x \uparrow 1} \frac{x-1}{|x-1|}$ f) $\lim_{x \to 1} \frac{x-1}{|x-1|}$ g) $\lim_{x \to 4^+} \sqrt{x-4}-2$ h) $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$

i) $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{x}$ j) $\lim_{x \to 3} \sqrt{x-5}$ k) $\lim_{x \to 0} \left(\frac{3+2x}{x+5x^2}-\frac{3}{x}\right)$

l) Re-write as a piece meal function: $F(x) = \frac{x+|x|}{|x|}$ m) Graph $F(x) = \frac{x+|x|}{|x|}$ n) $\lim_{x \to 0} \frac{x+|x|}{|x|}$

n) Explain (in English) ***exactly*** (no examples, please) what is meant by: $\lim_{x \to a} f(x) = L$. Be sure to include how "left-hand limits" , "right-hand limits", and "limits" are related and how they are different