Honors Analysis Name _____

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept***!***

Chapter 3 Exam

I Given $f(x) = \begin{cases} \frac{2}{3}x + 1 & -3 < x < -1 \\ 3 & x = -1 \\ -x - 2 & -1 < x < 1 \\ x - 4 & 1 < x < 3 \\ -\frac{4}{3}x + 3 & 3 \le x < 5 \end{cases}$ (20 pts tot)

a) Prove $f(x)$ is or is not continuous at $x = -1$

b) Prove $f(x)$ is or is not continuous at $x = 1$

c) Prove $f(x)$ is or is not continuous at $x = 3$

II *Using the definition of the derivative* which yields a function,

find $f'(x)$ given $f(x) = x^2 + 1$. (15 pts)

III Given $f(x) = \left|x^2 - 4\right|$. Using the first definition of the derivative,

PROVE $f'(2)$ does not exist. (15 pts)

IV Given $f(x) = 4x^3 - 12x^2 + 8$ (35 pts tot)

a) State the coordinates of all *x*-intercepts of $f(x)$.

b) State the intervals (using proper notation) where $f(x)$ is increasing; decreasing.

c) Identify and state the coordinates of all extrema. **Justify**.

d) State the coordinates of any Points of Inflection. **Justify**.

e) Draw an accurate graph of $f(x)$. Specifically show all coordinates found above.

V Find the following limits. (If they do not exist, explain fully how you know that.) (5 pts ea)

a) $\lim_{x \to 1} \frac{x+1}{x^2 - 1}$ b) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$ c) $\lim_{x \to 3} \frac{\sqrt{x-1}}{\sqrt{x-5}}$

Extra Credit ---------- 5 pts ----------

A square of perimeter 20 is inscribed in a square of perimeter 28.
What is the maximum distance between a vertex of the inner square and a vertex of the outer square?