

# Honors Analysis

Name \_\_\_\_\_

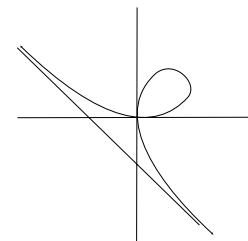
Copy original problem.

Per \_\_\_\_\_

Date \_\_\_\_\_

Convince *me* that **you** understand the concept!*No decimal fractions.*

## Chapter 4 Mechanical Exam

**I**The curve on the right is called the *folium of Descartes* and has the equation: $x^3 + y^3 = 3xy$ . The curve was originally proposed by Rene Descartes as a challenge to Pierre Fermat to find its tangent line.The curve has as a tilted asymptote  $y = -x - 1$ .

(25 pts)

- Find that tangent line at  $(\frac{2}{3}, \frac{4}{3})$ . Write the equation of the line in *standard* form, ie. without fractions and equal to zero.
- Find the coordinate where the tangent line in part (a) intersects with the asymptote.

**II**Given  $f(x) = \frac{1}{1-x}$ .

(tot 30 pts)

- Find the domain and range of  $f(x)$ .
- Determine the  $x$ -intercept of the tangent line to  $f(x)$  at  $x = 3$ .
- Find the equation(s) of all tangent lines to  $f(x)$  which are perpendicular to:  $x + 4y - 2 = 0$

**III**Identify (and justify) the coordinates of all extreme values.  $f(x) = (3x - 2)^2(4 - 3x)^3$  for  $x \in [-1, 2]$ .

(20pts)

**IV**Given  $f(x)$ . Determine  $f'(x)$ . Do not simplify your answers. (for example, leave negative exponents as negative exponents, do not combine constants, etc.) Basically, I am looking for evidence that you have used the proper formula in the proper order at the proper time. (15 ps)

$$f(x) = \frac{(4x - 3\sqrt{x})^3 \sqrt{3x^2 + 2x + 1}}{3x^2 \sqrt{2x^2 - 3x + 1}}$$

**V**Newton's Method. Explain how "Newton's Method for finding roots" of equations works. Do *not* use specific examples. (You *are* allowed to use coordinates like  $(x_1, f(x_1))$ ). Conclude your explanation with "Newton's recursion formula". (10 pts)**Extra Credit** ----- 5 pts -----Reconsider the folium of Descartes in section I. What is the shortest distance from the origin to the line tangent to the folium at  $(\frac{2}{3}, \frac{4}{3})$ ?