

**Honors Analysis**

Name \_\_\_\_\_

Copy the original problem.

Convince *me* that **you** understand the concept.

Per \_\_\_\_\_ Date \_\_\_\_\_

**No Calculators!****CHAPTER # 3**

**I** Given:  $f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ 3-(x-2)^2 & \text{if } x \geq 1 \end{cases}$  (tot 20 pts)

- a) Prove  $f(x)$  is continuous at  $x = 1$ .                      b) Prove  $f'(1)$  does not exist.

**II** Using the second definition of the derivative ( $\lim_{h \rightarrow 0}$  method), find  $f'(x)$  when  $f(x) = x + \frac{1}{x}$  (tot 20 pts)

**III** Given:  $f(x) = x^4 - 4x^3 + 10$  (tot 40 pts)

- a) Find  $f'(x)$  and  $f''(x)$ .
- b) Draw and properly label areas of interest on number lines for each  $f'(x)$  and  $f''(x)$ .
- c) Find the coordinates of the critical points of  $f(x)$ . Identify them as max, min or PI. Justify your responses.
- d) Are there any PI which are not at critical values of  $x$ ? Where?
- e) Determine and state the interval(s) where:
- 1)  $f(x)$  is increasing.                      2)  $f(x)$  is concave up.
- f) Sketch  $f(x)$ .

**IV** Determine the following limits: (5 pts ea)

- a)  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$                       b)  $\lim_{x \rightarrow 0} \left( \frac{3+2x}{x+5x^2} - \frac{3}{x} \right)$
- c)  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt[3]{x}-1}$                       d)  $\lim_{x \rightarrow -1} \frac{x+1}{x^2+1}$

**EXTRA CREDIT** ===== 5 pts =====

Given  $f(x) = |x+1| \operatorname{sgn}(x-2)$ . Sketch  $f(x)$ . Prove  $f(x)$  is or is not continuous at  $x = 2$ .