Key

Advanced Placement Calculus AB
1st Semester Final Exam

No Calculators Allowed

This exam is composed of 50 multiple choice questions. Each question carries a 2.3 point value so the maximum available points is 115. That is 100 points plus 15 points of extra credit.

Do not write or mark in any manner on this copy of the final exam.

On your scrantron, include your name *and* your exam number. You will need to remember your exam number. I will post exam results by exam number and will not give out exam numbers.

There is no partial credit (as is usual on a multiple choice exam) however there is something to be learned from any errors which are made. For that reason, you are encouraged to do your work in a neat and orderly manner. Be sure to put the problem number with each problem you do for later reference. Staple your scratch paper in the proper order and turn in when you turn in your exam and scrantron sheet. Your name must be written at the top of each page of scratch paper you use.

There is no penalty for wrong answers (as there is on the AP Calculus test, S.A.T. , etc.)

1. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

a) $y = -6x - 6$ b) $y = -3x + 1$ c) $y = 2x + 10$ d) $y = 3x - 1$ e) $y = 4x + 1$

2. If $f(x) = (2x + 1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

a) 0 b) 24 c) 48 d) 240 e) 384

3. If $y = \frac{3}{4 + x^2}$, then $\frac{dy}{dx} =$

a) $\frac{-6x}{\left(4 + x^2\right)^2}$ b) $\frac{3x}{\left(4 + x^2\right)^2}$ c) $\frac{6x}{\left(4 + x^2\right)^2}$ d) $\frac{-3}{\left(4 + x^2\right)^2}$ e) $\frac{3}{2x}$

4. If $x = 2t - 7$ and $y = 8t^2$, then $\frac{d^2y}{dx^2} =$

a) $8t$ b) $16t$ c) 4 d) 8 e) $4t$

5. $\lim_{n \to \infty} \frac{4n^2}{n^2 + 10000n}$

a) 0 b) $\frac{1}{2500}$ c) 1 d) 4 e) nonexistent

6. If $f(x) = x$, then $f'(5) =$

a) 0 b) $\frac{1}{5}$ c) 1 d) 5 e) $\frac{25}{2}$

7. If $\lim_{x \to a} f(x) = L$ is a real number, which of the following must be true?

a) $f'(a)$ exists. b) $f(x)$ is continuous at $x = a$.

c) $f(x)$ is defined at $x = a$. d) $f(a) = L$ e) None of the above.

8. The slope of the line tangent to the graph of $y = \ln \frac{x}{2}$ at $x = 4$ is

a) $\frac{1}{8}$ b) $\frac{1}{4}$ c) $\frac{1}{2}$ d) 1 e) 4

9. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

a) -1 only b) 0 only c) 1 only d) -1 and 1 only e) $-1, 0$, and 1

10. If $y = 10^{x^2-1}$, then $\frac{dy}{dx} =$

a) $(\ln 10)\, 10^{x^2-1}$ b) $2x\, 10^{x^2-1}$ c) $(x^2 - 1)\, 10^{x^2-2}$

d) $2x(\ln 10)\, 10^{x^2-1}$ e) $x^2(\ln 10)\, 10^{x^2-1}$

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

a) 0 b) 2 c) 4 d) 8 e) 12

12. If $f(g(x)) = \ln(x^2 + 4)$, $f(x) = \ln x^2$, and $g(x) > 0$ for all real x then $g(x) =$

a) $\frac{1}{\sqrt{x^2 + 4}}$ b) $\frac{1}{x^2 + 4}$ c) $\sqrt{x^2 + 4}$ d) $x^2 + 4$ e) $x + 2$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

a) $-\frac{2x + y}{x + 3y^2}$ b) $-\frac{x + 3y^2}{2x + y}$ c) $\frac{-2x}{1 + 3y^2}$ d) $\frac{-2x}{x + 3y^2}$ e) $-\frac{2x + y}{x + 3y^2 - 1}$

14. If $x^2 + 2y = 20$ and $\frac{dx}{dt} = -6$, find $\frac{dy}{dt}$ when $x = 4$.

a) 4 b) -2 c) -6 d) 24 e) 8

15. The domain of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers x such that

a) $|x| < 2$ b) $|x| \le 2$ c) $|x| > 2$ d) $|x| \ge 2$ e) x is a real number

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

a) -2 b) 0 c) 1 d) 2 e) 4

17. $\lim_{x \to 0}(x \csc x) =$

a) $-\infty$ b) -1 c) 0 d) 1 e) ∞

18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

a) -1 b) 0 c) $-2\sin(2x)$ d) $-2(\cos x + \sin x)$ e) $-2(\cos x - \sin x)$

19. $\lim_{x \to 0} x^2 \ln x =$

a) 0 b) 1 c) 2 d) -2 e) DNE

20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$

a) $\frac{-\sin x}{1 + \cos^2 x}$ b) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ c) $(\operatorname{arcsec}(\cos x))^2$

d) $\frac{1}{1 + \arccos^2 x}$ e) $\frac{1}{1 + \cos^2 x}$

21. If the domain of the function f given by $f(x) = \frac{1}{1 - x^2}$ is $\{x : |x| > 1\}$

what is the range of f ?

a) $f(x) \,\varepsilon\, (-\infty, -1)$ b) $f(x) \,\varepsilon\, (-\infty, 0)$ c) $f(x) \,\varepsilon\, (-\infty, 1)$

d) $f(x) \,\varepsilon\, (-1, \infty)$ e) $f(x) \,\varepsilon\, (0, \infty)$

22. If $f(x) = \frac{\ln x}{x}$ for $x > 0$, which of the following is true?

a) f is increasing for all x greater than 0. b) f is increasing for all x greater than 1.

c) f is decreasing for all x between 0 and 1. d) f is decreasing for all x between 1 and e

e) f is decreasing for all x greater than e.

23. $\frac{d}{dx}\left(\frac{1}{x^3} - \frac{1}{x} + x^2\right)$ at $x = -1$ is

a) -6 b) -4 c) 0 d) 2 e) 6

24. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.

II. The maximum value of $|f(x)|$ is 7.

III. The minimum value of $f(|x|)$ is 0.

a) I only b) II only c) I and II only d) II and III only e) I, II, and III

25. If $f(x) = e^x$ which of the following is equal to $f'(e)$?

a) $\lim\limits_{h \to 0} \dfrac{e^{x+h}}{h}$ b) $\lim\limits_{h \to 0} \dfrac{e^{x+h} - e^e}{h}$ c) $\lim\limits_{h \to 0} \dfrac{e^{x+h} - e}{h}$ d) $\lim\limits_{h \to 0} \dfrac{e^{x+h} - 1}{h}$ e) $\lim\limits_{h \to 0} \dfrac{e^{e+h} - e^e}{h}$

26. The graph of $y^2 = x^2 + 9$ is symmetric with respect to which of the following?

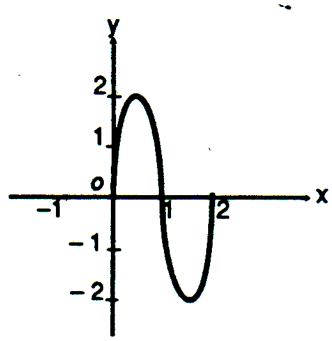
I. The x-axis II. The y-axis III. The origin

a) I only b) II only c) I and II only d) II and III only e) I, II, and III

27. Which of the following functions are continuous for all real numbers x?

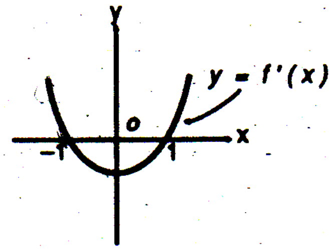
I. $y = x^{2/3}$ II. $y = e^x$ III. $y = \tan x$

a) None b) I only c) II only d) I and II only e) I and III

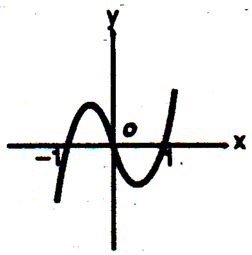


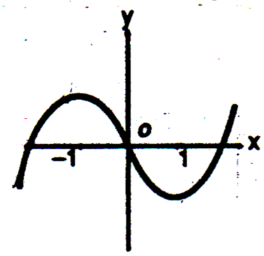
28. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

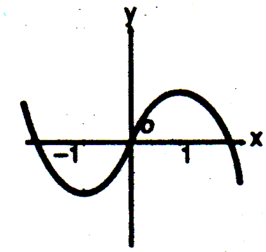
a) $y = 2\sin\left(\dfrac{\pi x}{2}\right)$ b) $y = \sin(\pi x)$ c) $y = \sin(2x)$ d) $y = 2\sin(\pi x)$ e) $y = 2\sin(2x)$

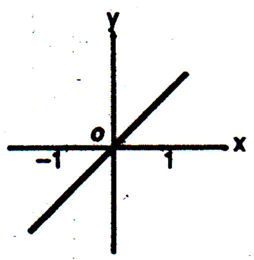


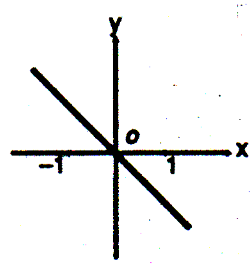
29. The graph of the <u>derivative</u> of f is shown in the figure above. Which of the following could be the graph of f?

a) 

b) 

c) 

d) 

e) 

30. If $f(x) = \dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

a) 2 b) $\dfrac{1}{2}$ c) $1 + \dfrac{\pi}{2}$ d) $\dfrac{\pi}{2} - 1$ e) $1 - \dfrac{\pi}{2}$

31. If $xy^2 + 2xy = 8$, then at the point $(1, 2)$, $y' =$

a) $-\dfrac{5}{2}$ b) $-\dfrac{4}{3}$ c) -1 d) $-\dfrac{1}{2}$ e) 0

32. $\dfrac{d}{dx}\left(\ln \dfrac{1}{1-x}\right) =$

a) $\dfrac{1}{1-x}$ b) $\dfrac{1}{x-1}$ c) $1 - x$ d) $x - 1$ e) $(1-x)^2$

33. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

a) 0 only b) 2 only c) 3 only d) 0 and 3 e) 2 and 3

34. Which of the following functions shows that the statement "If a function is continuous at $x = 0$, then it is differentiable at $x = 0$" is false?

a) $f(x) = x^{-\frac{4}{3}}$ b) $f(x) = x^{-\frac{1}{3}}$ c) $f(x) = x^{\frac{1}{3}}$ d) $f(x) = x^{\frac{4}{3}}$ e) $f(x) = x^{3}$

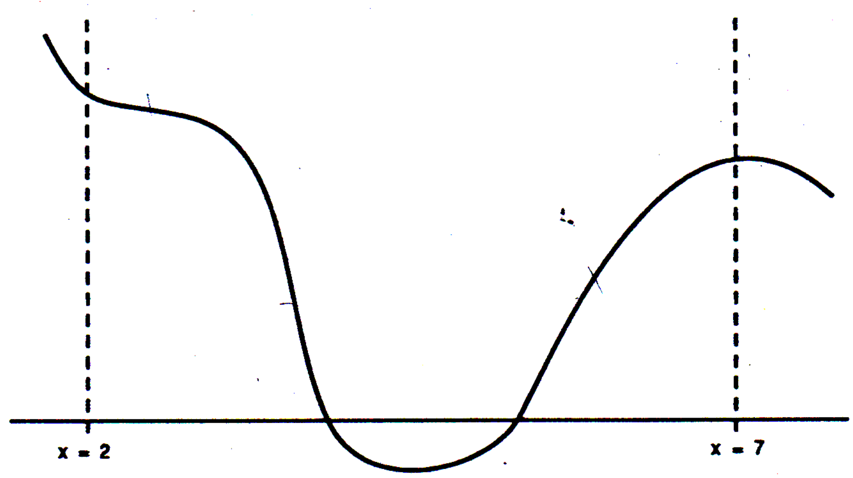
35. If $f(x) = x \ln x^2$, then $f'(x) =$

a) $\ln x^2 + 1$ b) $\ln x^2 + 2$ c) $\ln x^2 + \frac{1}{x}$ d) $\frac{1}{x^2}$ e) $\frac{1}{x}$

36. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

a) $f'(x) + f''(x)$ b) $f'(x) + \left(f''(x)\right)^2$ c) $\left(f'(x) + f''(x)\right)^2$

d) $\left(f'(x)\right)^2 + f''(x)$ e) $2f'(x) + f''(x)$



37. The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

a) one b) two c) three d) four e) five

38. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

a) 6 b) 8 c) 16 d) $4\sqrt{3}$ e) $12\sqrt{3}$

39. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

a) no values b) 0 only c) $\frac{1}{2}$ only d) 1 only e) 0 and $\frac{1}{2}$

40. For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx} =$

a) $x \ln(\sin x)$ b) $(\sin x)^x \cot x$ c) $x(\sin x)^{x-1} \cos x$

d) $(\sin x)^x[x \cos x + \sin x]$ e) $(\sin x)^x[x \cot x + \ln(\sin x)]$

41. $\lim\limits_{x \to \pi/4} \dfrac{\sin(x - \pi/4)}{x - \pi/4} =$

a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{\pi}{4}$ d) 1 e) nonexistent

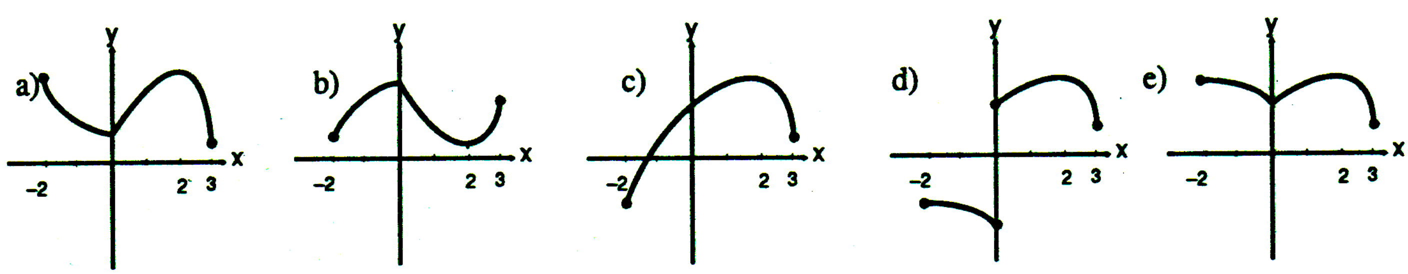
42. If $x = t^3 - t$ and $y = \sqrt{3t + 1}$, then $\frac{dy}{dx}$ at $t = 1$ is

a) $\frac{1}{8}$ b) $\frac{3}{8}$ c) $\frac{3}{4}$ d) $\frac{8}{3}$ e) 8

43. An equation of the line normal to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

a) $4x + y = 10$ b) $x - 4y = 23$ c) $4x - y = 2$ d) $x + 4y = 25$ e) $x + 4y = -25$

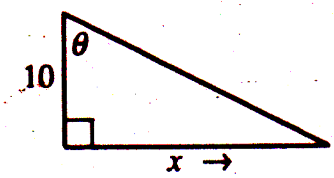
44. Let f be a function that is continuous on the close interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



45. If the graph of $y = ax^3 + 4x^2 + cx + d$ has a point of inflection at $(1, 0)$, then the value of a is

a) 2 b) $-\frac{4}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{3}$ e) $\frac{9}{7}$

46. Consider the figure on the right where the distance x is increasing at the rate of 50 units per second. In radians per second, what is the rate of change of the angle θ when $x = 10$?

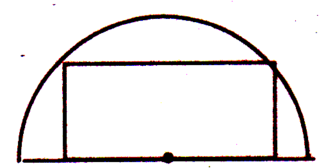


a) 1 b) 1.25 c) 1.5 d) 2 e) 2.5

47. If $g(x)$ is a second degree polynomial satisfying
$g(0) = 3$, $g'(2) = 10$ and $g''(10) = 4$, then $g(x) =$

a) $3x^2 + 3$ b) $x^2 + 4x + 3$ c) $2x^2 + 4x + 3$ d) $2x^2 + 2x + 3$ e) $x^2 + 6x + 3$

48. The figure shown is a rectangle enclosed by a semicircle of radius 1 foot. What is the maximum area of such a rectangle?



a) 2 sq ft b) 1.5 sq ft c) 1.25 sq ft d) 1 sq ft e) 0.75 sq ft

49. What is the absolute minimum value of the function $g(x) = xe^x$?

a) -1 b) $-e$ c) $-\frac{1}{e}$ d) $-\frac{1}{e^2}$ e) $-\frac{1}{2}$

50. The graph of $y = 2x^3 + 5x^2 - 6x + 7$ has a point of inflection at $x =$

a) $-\frac{5}{3}$ b) 0 c) $-\frac{5}{6}$ d) $\frac{5}{2}$ e) -2

① $Y = x^3 + 3x^2 + 2$

$y' = 3x^2 + 6x$

$y'' = 6x + 6 \overset{set}{=} 0$

$x = -1$

$y\big|_{x=-1} = -1 + 3 + 2 = 4$

$y'\big|_{x=-1} = 3 - 6 = -3$

$m = -3 \quad (-1, 4)$

$y - 4 = -3(x + 1)$

$y = -3x + 1$

(B)

② $f(x) = (2x+1)^4$

1st $4(2x+1)^3 \cdot 2$

2nd $12(2x+1)^2 \cdot 2 \cdot 2$

3rd $24(2x+1)(2 \cdot 2 \cdot 2)$

4th $24 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$16 \cdot 24$

384

(E)

③ $y = \dfrac{3}{4 + x^2}$

$y' = \dfrac{-6x}{(4 + x^2)^2}$

(A)

④ $x = 2t - 7$

$y = 8t^2$

$\dfrac{dx}{dt} = 2$

$\dfrac{dy}{dt} = 16t$

$\dfrac{dy}{dx} = 8t$

$\dfrac{d^2y}{dx^2} = 8\dfrac{dt}{dx}$

$= \dfrac{8}{2} = 4$

(C)

⑤ $\lim\limits_{N \to \infty} \dfrac{4N^2}{N^2 + 100000N} \quad \dfrac{\frac{1}{N^2}}{\frac{1}{N^2}}$

4

(D)

⑥ $f(x) = x$

$f'(x) = 1$

$f'(5) = 1$

(C)

⑦ (E) None of These

If the limit exists the others may or may not be true

a) Der b) cont c) cont D) cont

(8) $Y = \ln\frac{x}{2}$

$\ln x - \ln 2$

$y' = \frac{1}{x}\Big|_{x=4} = \frac{1}{4}$ (B)

(9) $f(x) = 3x^5 - 5x^3 + 15$

$f'(x) = 15x^4 - 15x^2$

$15x^2(x^2-1)$

$+\ 0\ -\ 0\ -\ 0\ +$

$-1 \quad 0 \quad 1$

max only at $x=-1$ (A)

(10) $Y = 10^{x^2-1}$

$y' = \ln 10\,[2x]\,10^{x^2-1}$

D

(11) $S(t) = t^2 + 4t + 4$

$s' = v = 2t + 4$

$s'' = a = 2\Big|_{t=4} = 2$ (B)

(12) $f(g(x)) = \ln(x^2+4)$

$f(x) = \ln x^2$

$(g(x))^2 = x^2 + 4$

$g(x) = \sqrt{x^2+4}$

(C)

(13) $x^2 + xy + y^3 = 0$

$2x + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x+y}{x+3y^2}$ (A)

(14) $x^2 + 2y = 20 \qquad \frac{dx}{dt} = -6$

$2x\frac{dx}{dt} + 2\frac{dy}{dt} = 0$

$\cancel{2}\cdot 4(-6) + \cancel{2}\frac{dy}{dt} = 0$

$\frac{dy}{dt} = 24$

(D)

(15) $f(x) = \ln(x^2-4)$

$x^2 - 4 > 0$

$(x-2)(x+2)$

$+\ 0\ -\ 0\ +$

$-2 \quad 0 \quad 2$

$D_f\ |x| > 2$ C

(16) $~~f(x)=~~ x^3-3x^2$

$f'(x)=3x^2-6x$

$3x(x-2)\overset{\text{set}}{=}0$

$+\ 0\ -\ 0\ +$

$0 \quad 2$

max at $x=0$ (B)

f' test.

(17) $\lim_{x\to 0} x\csc x$

$\lim_{x\to 0} \frac{x}{\sin x}=1$ (D)

(18) $y=\cos^2 x-\sin^2 x$

$=\cos 2x$

$y'=-2\sin 2x$ (C)

(19) $\lim_{x\to 0} x^2\ln x$

$\lim_{x\to 0} \frac{\ln x}{\frac{1}{x^2}} \quad \frac{\infty}{\infty}$

$\lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$

$\lim_{x\to 0} -\frac{x^2}{2}=0$ (A)

(20) $y=\tan^{-1}(\cos x)$ (A)

$\frac{dy}{dx}=\frac{1(-\sin x)}{1+\cos^2 x}$

(21) $f(x)=\frac{1}{1-x^2} \quad |x|>1$

$\frac{\text{top} +}{\text{bottom} -}=$ Negative

$\lim_{x\to \pm 1} f(x)\to -\infty$ ∴ Range $(-\infty, 0)$

$\lim_{x\to \pm\infty} f(x)\to 0$ (B)

(22) $f(x)=\frac{\ln x}{x} \quad \frac{u}{v}$

$f'(x)=\frac{x\cdot\frac{1}{x}-\ln x}{x^2}$

$=\frac{1-\ln x}{x^2}$

$+\ \otimes\ +\ 0\ -$

$0 \quad e$

E

(23) $\frac{d}{dx}\left(\frac{1}{x^3}-\frac{1}{x}+x^2\right)\Big|_{x=-1}$

$-\frac{3}{x^4}+\frac{1}{x^2}+2x\Big|_{x=-1}=$

$-3+1-2=-4$

(B)

(24) $-7 < f(x) < 5$

I $f(\text{ANY}) \overset{\text{MAX}}{=} 5$

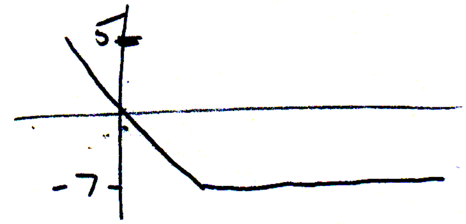
$|f(x)| = 7$

II only (B)

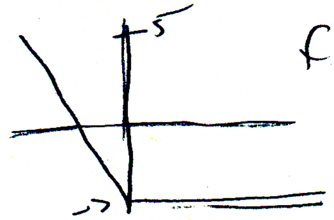
I maybe

II True

III No



$-7 < f(\text{Pos}) < 0$ so I false



$f(0) = -7$ so min Not at 0

(25) $f(x) = e^x$

$f'(e) = e^e$

$\lim_{h \to 0} \dfrac{e^{e+h} - e^e}{h}$

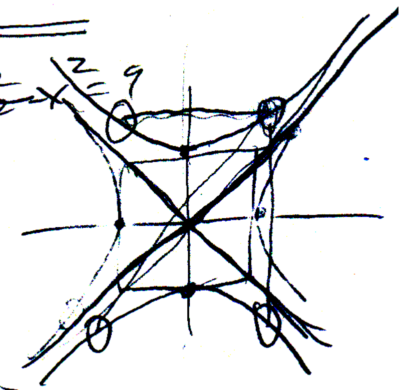
(E)

(26) $y^2 = x^2 + 9$

I Yes $(-y) = y$

even II Yes $(-x) = x$

III Yes $(-y) \to y$ $(-x) \to x$



$y^2 - x^2 = 9$

(E)

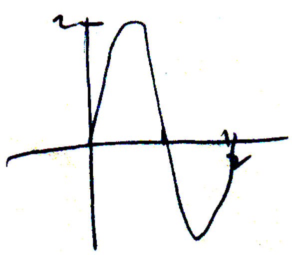
(27) $y = x^{\frac{2}{3}}$ Yes

$y = e^x$ Yes

$y = \tan x$ No

cont? I, II (D)

(28)



Per 2 $\frac{2\pi}{b} = 2$

Amp 2 $\pi = b$

Sine

$y = 2\sin \pi x$

(D)

(29) f' + 0 − 0 + ; −1, 3, 1

(B)

(30) $f(x) = \dfrac{x}{\tan x}$

$$\left. \frac{\tan x - x\sec^2 x}{\tan^2 x} \right|_{x = \frac{\pi}{4}} =$$

$$\frac{1 - \frac{\pi}{4} \cdot 2}{1}$$

$1 - \frac{\pi}{2}$ E

31 $xy^2 + 2xy = 8$

$y^2 + 2xyy' + 2y + 2xy' = 0$

$y' = \dfrac{-(y^2+2y)}{2xy+2x}$

(1, 2)

$\dfrac{-(4+4)}{4+2} = \dfrac{-8}{6} = \dfrac{-4}{3}$ B

32 $\dfrac{d}{dx}\left(\ln\left(\dfrac{1}{1-x}\right)\right)$

$\ln 1 - \ln(1-x)$

$y' = \dfrac{+1}{1-x}$ A

33 $f(x) = x^3 - 3x^2$

$f(0) = 0$

$f(3) = 0$

$f'(x) = 3x^2 - 6x \overset{\text{set}}{=} 0$

$3x(x-2) = 0$

only $x>2$. B

because $\overline{X}$ is in open interval.

34 a) No not cont at x=0

b) No "

c) Yes

d) No Looks true

E) No "

C

35 $f(x) = x \ln x^2$

$= 2x \ln x$

$f' = 2\ln x + 2$

$\ln x^2 + 2$

B

36 $g(x) = e^{f(x)}$

$g'(x) = f'(x)e^{f(x)}$

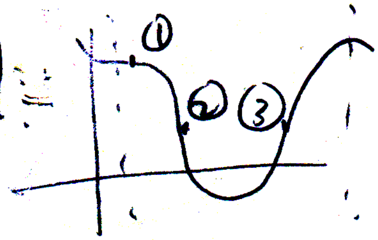
$g''(x) = e^{f(x)}f''(x) + (f'(x))^2 e^{f(x)}$

$h(x)e^{f(x)} \overset{\text{set}}{=} [f''(x) + (f'(x))^2]e^{f(x)}$

$h(x) = f''(x) + (f'(x))^2$

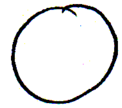
D

(37)



(C)

(38)



$A = \pi r^2 \longrightarrow 64\pi \overset{set}{=} \pi r^2$

$r = 8$

$\frac{dA}{dt} = 96\pi$

$\frac{dr}{dt}\Big|_{A=64} = ?$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$96\pi = 2\pi(8)\frac{dr}{dt}$

$6 = \frac{dr}{dt}$ (A)

(39) $x(t) = \overset{u}{t}\,\overset{v}{e^{-2t}}$

$\dot{x} = e^{-2t} - 2te^{-2t}$

$= e^{-2t}[1 - 2t] \overset{set}{=} 0$

$t = \frac{1}{2}$

(C)

(40) $y = (\sin x)^x$

$\ln y = \overset{u}{x}\,\overset{v}{\ln \sin x}$

$\frac{1}{y}\frac{dy}{dx} = \ln \sin x + \frac{x}{\sin x}\cdot\cos x$

$\frac{dy}{dx} = (\sin x)^x\left[\ln(\sin x) + x\cot x\right]$

(E)

(41) $\lim\limits_{x \to \frac{\pi}{4}} \frac{\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$

let $\theta = x - \frac{\pi}{4}$

$\lim\limits_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (D)

(42) $x = t^3 - t \qquad y = \sqrt{3t+1}$

$\frac{dx}{dt} = 3t^2 - 1 \qquad \frac{dy}{dt} = \frac{3}{2\sqrt{3t+1}}$

$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = \frac{3}{2\cdot 2} = \frac{3}{4}$

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx}$

$\frac{dy}{dx} = \frac{3}{8}$ (B)

(43) $y = x^3 + 3x^2 + 7x - 1$

$y' = 3x^2 + 6x + 7\Big|_{x=-1} = 3 - 6 + 7 = 4$

$\perp$ so $m = -\frac{1}{4}$

$y\Big|_{x=-1} = -1 + 3 - 7 - 1 = -6$

$(-1, -6)$ $m = -\frac{1}{4}$

$y + 6 = -\frac{1}{4}(x+1)$

$= -\frac{1}{4}x - \frac{1}{4}$

$y = -\frac{1}{4}x - \frac{25}{4}$

$4y = -x - 25$

$x + 4y = -25$

(E)

(44) A) No concave up $x<0$ B) No concave up $x>0$ c) No Der Exists at $x=0$ D) No f Not cont E) Yes!

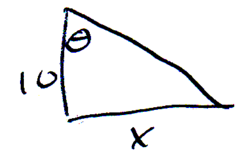
(45) $y = ax^3 + 4x^2 + cx + d$

$y' = 3ax^2 + 8x + c$

$y'' = 6ax + 8\Big|_1 = 6a + 8 \overset{set}{=} 0$

$a = -\frac{4}{3}$ (B)

(46) $\frac{dx}{dt} = 50$



$\frac{d\theta}{dt} = ?$

$\frac{x}{10} = \tan\theta$

$\frac{1}{10}\frac{dx}{dt} = \sec^2\theta \frac{d\theta}{dt}$

$\frac{1}{10} \cdot 50 =$

$5\cos^2\theta = \frac{d\theta}{dt}\Big| = \frac{5}{2}$

$x = 10$

$\theta = \frac{\pi}{4}$ (E)

(47) $g(0) = 3$

$g'(2) = 10$

$g''(10) = 4$

$g(x) = ax^2 + bx + c$

$g(0) = c \overset{set}{=} 3 \quad c = 3$

$g'(x) = 2ax + b =$

$4a + b = 10$

$g''(x) = 2a = 4$

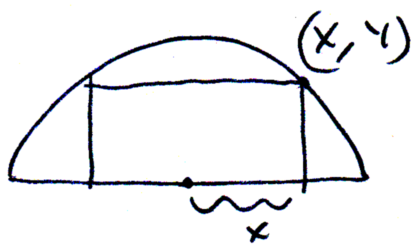
$a = 2$

$8 + b = 10$

$b = 2$

$g = 2x^2 + 2x + 3$ (D)

(48)



$x^2 + y^2 = 1$

$y = \sqrt{1-x^2}$

$y = \frac{\sqrt{2}}{2}$

Area $= 2xy$

$A(x) = 2x\sqrt{1-x^2}$

$A' = 2\sqrt{1-x^2} + \frac{\cancel{2}x(-2x)}{\cancel{2}\sqrt{1-x^2}}$

$\frac{2(1-x^2) - 2x^2}{\sqrt{1-x^2}}$

$\frac{2-4x^2}{\sqrt{1-x^2}} \overset{set}{=} 0$

$4x^2 = 2$

$x = \sqrt{\frac{1}{2}}$

$\frac{\sqrt{2}}{2}$

f' + 0 −

$\frac{1}{\sqrt{2}}$

max f' ~~(cont)~~

$A = 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 1$

D

(49)

$g(x) = \overset{u}{x}\,\overset{v}{e^{x}}$

$g'(x) = e^{x} + xe^{x}$

$e^{x}(1+x) \overset{set}{=} 0$

g': $-$ 0 $+$ at -1 ; 3

abs min at $x = -1$

$-\frac{1}{e}$

(c)

(50)

$y = 2x^{3} + 5x^{2} - 6x + 7$

$y' = 6x^{2} + 10x - 6$

$y'' = 12x + 10 \overset{set}{=} 0$

$x = \frac{-5}{6}$

(c)

$-$ 0 $+$ at $\frac{-5}{6}$; 0

IT Does change concavity.